Weighted DL-Lite in a possibility theory setting

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Weighted DL-Lite

- Representation
- Fusion
- Inconsistency
- Revision

Context
DL-Lite in few words

- Tradeoff between expressive power and complexity of reasoning
- Captures basic features of both ontology languages and conceptual modeling formalisms
- Provides the foundations of OWL2-QL especially dedicated to applications that use huge volumes of data
Outline

1. Possibilistic DL-Lite: Representation

2. Possibilistic DL-Lite: Inconsistency management

3. Possibilistic DL-Lite: Revision and Fusion
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1. Possibilistic DL-Lite: Representation
2. Possibilistic DL-Lite: Inconsistency management
3. Possibilistic DL-Lite: Revision and Fusion
Starting point

For representing a domain of interest, we use:

- \( a \): an individual or a constant.
- \( A \): an atomic concept to denote a sets of individuals.
- \( P \): an atomic role to denote binary relation among individuals.
- \( \alpha \in ]0, 1] \): to denote the uncertainty degree of an axiom.
- \( \mathcal{K} = \langle \mathcal{T}, \mathcal{A} \rangle \): to denote a knowledge base where
  - \( \mathcal{T} \) is a TBox
  - \( \mathcal{A} \) is an ABox
π-DL-Lite knowledge base: the ABox

An ABox consists of a set of:

- membership assertions on atomic concepts
  \[ \langle A(a), \alpha \rangle \]

- membership assertions on atomic roles
  \[ \langle P(a, b), \alpha \rangle \]
A description language is characterized by a set of constructs used for building complex concepts and roles form atomic ones.

- “−” to denote the inverse of role: if $P = (a, b)$ then $P^- = (b, a)$.
- “¬” to denote the negation of a concept or role.
- “∃” to denote the first element of a role or an inverse of role.
An $DL$-$Lite_R$ TBox consists of a set of:

- Concept inclusion assertion with $B \rightarrow A \mid \exists P \mid \exists P^-$:

  $$\langle B_1 \sqsubseteq B_2, \alpha \rangle \quad \langle B_1 \sqsubseteq \neg B_2, \alpha \rangle$$

- Roles inclusion assertions with $R \rightarrow P \mid P^-$:

  $$\langle R_1 \sqsubseteq R_2, \alpha \rangle \quad \langle R_1 \sqsubseteq \neg R_2, \alpha \rangle$$
The semantics is given in terms of **interpretations**.

An interpretation $I = (\Delta, .^I)$ consists of

- a non-empty domain (set) $\Delta$
- an interpretation function $^I$, which assigns:
  - to each individual $a$ an element $a^I \in \Delta^I$,
  - to each atomic concept $A$ a subset $A^I \subseteq \Delta^I$
  - to each atomic role $P$ a subset $P^I \subseteq \Delta^I \times \Delta^I$

The $^I$ is extended to complex concepts and roles according to their structure.

- $(\neg A)^I = \Delta^I \setminus A^I$
- $(P^-)^I = \{(y, x) \in \Delta^I \times \Delta^I : (x, y) \in P^I\}$
- $(\exists R)^I = \{x \in \Delta^I : \exists y \in \Delta^I. (x, y) \in R^I\}$
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The semantics is given by specifying whether $I$ satisfies or not assertions:

**TBox**
- $I \models B \sqsubseteq C$ iff $B^I \subseteq C^I$
- $I \models R \sqsubseteq E$ iff $R^I \subseteq E^I$

**ABox**
- $I \models A(a_i)$ iff $a_i^I \in A^I$
- $I \models P(a_i, a_j)$ iff $(a_i^I, a_j^I) \in P^I$

**Model of a DL knowledge base**

An interpretation $I$ is a model of $\mathcal{K} = \{\mathcal{T}, \mathcal{A}\}$, written $I \models \mathcal{K}$ if it satisfies every assertion in $\mathcal{T}$ and $\mathcal{A}$.
A possibility distribution $\pi : \Omega \rightarrow [0, 1]$ where $\Omega = \{ I = (\Delta, .') \}$

- $\pi (I) \in [0, 1]$ represents the consistency of $I$ relative to the available knowledge.

Let $\langle \phi, \alpha \rangle \in K$ a possibilistic axiom:
- if $I \models \phi$ then $\pi_K (I) = 1$.
- if $I \not\models \phi$ then $\pi_K (I) = 1 - \alpha$.

For every $I \in \Omega$, $\pi_K$ associated to $K$ a $\pi$-DL-Lite KB

$$\pi_K (I) = \begin{cases} 1 & \text{if } \forall \langle \phi_i, \alpha_i \rangle \in K, I \models \phi_i \\ 1 - \max \{ \alpha_i : (\phi_i, \alpha_i) \in K \mid I \not\models \phi_i \} & \text{otherwise} \end{cases}$$
Ranking interpretations

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Let $\langle \phi, \alpha \rangle \in \mathcal{K}$ a possibilistic axiom:

- if $I \models \phi$ then $\pi_{\mathcal{K}}(I) = 1$.
- if $I \not\models \phi$ then $\pi_{\mathcal{K}}(I) = 1 - \alpha$.

For every $I \in \Omega$, $\pi_{\mathcal{K}}$ associated to $\mathcal{K}$ a $\pi$-DL-Lite KB:

$$
\pi_{\mathcal{K}}(I) = \begin{cases} 
1 & \text{if } \forall \langle \phi_i, \alpha_i \rangle \in \mathcal{K}, I \models \phi_i \\
1 - \max \{ \alpha_i : (\phi_i, \alpha_i) \in \mathcal{K} | I \not\models \phi_i \} & \text{otherwise}
\end{cases}
$$
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Incoherence vs Inconsistency

In DLs, there are two similar concepts to reflect the presence of conflicts in KBs.

**Incoherence**
- A DL-Lite TBox $\mathcal{T}$ is said to be incoherent if there exist a concepts $C$ such that for every interpretation $I$ which is a model of $\mathcal{T}$, we have $C^I = \emptyset$.
- $\mathcal{T} = \{B_1 \sqsubseteq B_2, B_1 \sqsubseteq \neg B_2\}$, all models $I$ of $\mathcal{T}$ are such that $B_1^I = \emptyset$.

**Inconsistency: a stronger concept of incoherence**
- A KB $\mathcal{K} = \langle \mathcal{T}, \mathcal{A} \rangle$ is said to be inconsistent if it does not admit a model.
- An inconsistency is always defined with respect to ABox, since a TBox may be incoherent but never inconsistent.
Incoherence vs Inconsistency

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**Incoherence**

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**Inconsistency: a stronger concept of incoherence**

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Positive inclusions axioms vs Negative inclusions axioms

A possibilistic TBox $\mathcal{T} = \{PI, NI\}$ can be viewed as composed of:

- **positive inclusions (PIs) of the form:**
  - $\langle B_1 \sqsubseteq B_2, \alpha \rangle$
  - $\langle R_1 \sqsubseteq R_2, \alpha \rangle$

- **negative inclusions (NIs) of the form:**
  - $\langle B_1 \sqsubseteq \neg B_2, \alpha \rangle$
  - $\langle R_1 \sqsubseteq \neg R_2, \alpha \rangle$
Let $\langle B_1 \sqsubseteq B_2, \alpha_1 \rangle$ be a weighted positive inclusion axiom:

- $\langle B_1 (a), \alpha_2 \rangle$
- $\langle B_2 (a), \alpha_3 \rangle$

A KB $\mathcal{K} = \langle T, A \rangle$ with only positive inclusions axioms in its TBox is always satisfiable.
Negative inclusions axioms

Let $\langle B_1 \sqsubseteq \neg B_2, \alpha_1 \rangle$ be a weighted negative inclusion axiom

- $\langle B_1 (a), \alpha_2 \rangle$

- $\langle B_2 (a), \alpha_3 \rangle$

A KB $\mathcal{K} = \langle \mathcal{T}, \mathcal{A} \rangle$ with negative inclusions axioms may cause Inconsistency

A conflict necessary implies:

- One NI axiom

- One or two membership assertion
Inconsistency check?

TBox

Negated closure

Search for all contradictions

ABox

Generation of conflicts (Revision)

Compute inconsistency degree (possibility theory frameworks)
Negated closure in $\pi$-DL-Lite

At the beginning $\pi$-$neg (T)$ is set to an empty set.

- **Rule 1:** Let $T = \{IP, IN\}$ then $IN \subseteq \pi - neg (T)$.

- **Rule 2:** Let $T = \{IP, IN\}$ then $\langle funct R, \alpha \rangle \subseteq \pi - neg (T)$.

- **Rule 3:** If $\langle B_1 \sqsubseteq B_2, \alpha_1 \rangle \in T$:
  - if $\langle B_2 \sqsubseteq \neg B_3, \alpha_2 \rangle \in \pi - neg (T)$ or $\langle B_3 \sqsubseteq \neg B_2, \alpha_2 \rangle \in \pi - neg (T)$
  - then add $\langle B_1 \sqsubseteq \neg B_3, min(\alpha_1, \alpha_2) \rangle$ to $\pi - neg (T)$

- ....
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Revision Problem

Initial KB + New information = Inconsistent KB
The input

New information (The input)

- Membership assertion
- PI axiom
- NI axiom
How modify $K$ to become consistent with $N$?

In order to restore consistency of $K \cup N$:

- we remove a membership assertion from original ABox.

- we remove a PIs or a NIs axioms from original TBox.

- we remove axioms from both ABox or TBox.
Revision operation (1/2)

Case 1

The input:

Membership assertion

K={T,A}

Revision is polynomial

Case 2

The input:

P1 axiom

K={PIs,A}

No conflict
Case 3

The input:

PL axiom

NI axiom

K=\{\{PLs,NIls\},A\}

Generation of conflicts is polynomial

Revision ????
Generation of removed collections

A NI axiom

\[ A \sqsubseteq \neg B \]

Conflicts

\{A(a), B(a)\} \rightarrow \{A(b), B(b)\} \rightarrow \{A(c), B(c)\} \rightarrow \ldots

Collections

\{A(a), B(b), A(c)\} \rightarrow \{A(a), A(b), B(c)\} \rightarrow \ldots
Generation of removed sets

\[ A \subseteq \neg B \]

\[ C \subseteq \neg D \]

Removed collections:
- \{A(a), B(b), A(c)\}
- \{A(a), A(b), B(c)\}
- \{C(e), D(f)\}
- \{C(e), D(e)\}

Removed sets:
- \{A(a), B(b), A(c), C(e), D(f)\}
- \{A(a), A(b), B(c), C(e), D(f)\}
- \{A(a), B(b), A(c), C(e), D(f)\}
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