Research report on on getting rid of incommensurability of uncertainty frameworks (incommensurable)

Livrable D2.5

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Abstract

DL-Lite is one of the most important tractable fragment of DLs that provides a powerful framework to compactly encode available knowledge with a low computational complexity of the reasoning process. Recently, some approaches have been proposed for merging multiple and potentially conflicting pieces of information. Egalitarist fusion modes privilege solutions that minimize the (local) dissatisfaction of each agent (source, expert) who is involved in the fusion process. This paper investigates egalitarist fusion of incommensurable ranked *DL-Lite* knowledge bases. operator are studied.

1 Introduction

In many situations, relevant informations are provided by different sources. Taking advantage of the different sources of information usually requires to perform some combination operation on the pieces of information, and leads to the data fusion problem.

Recently, some approaches have been proposed for merging possibly contradictory Description Logics knowledge bases (*e.g.* [8, 3, 2]). Knowledge bases can be either flat (no priority relation is provided between different formulas) or ranked. A ranked knowledge base (or a stratified knowledge base, a weighted belief base) is a set of well-founded formulas, each associated with a rank (assumed here to be an integer). The higher is the rank associated with a formula, the more important is the formula. In fact, ranked belief bases are convenient representations of what is usually known as epistemic states [5]. Namely, each ranked belief base induces a ranking on set of possible interpretations (or solutions). Interpretations assigned to lower ranks are considered to be more plausible than interpretations assigned to higher ranks. In particular, interpretations with the rank '0' are the most preferred ones and represent agent's current beliefs.

In this paper, the term beliefs is used since pieces of information provided by sources are uncertain. We will reserve the term constraints to completely sure and consistent pieces of information. Constraints should be present in the result of the fusion process, while beliefs can be accepted, weakened or if necessary ignored in the fusion process.

Among existing merging approaches, we can distinguish two important ones: utilitarist approaches (or majority approaches) and egalitarist approaches. Examples of majority approaches are those based on the "sum" operator, and examples of egalitarist approaches are those based on the maximum operator (and its extension Gmax), see [7] for more details. Given a set of n consistent belief bases (or preferences sets), provided by n sources (or agents, or experts), a majority approach tries to minimize a global dissatisfaction. In particular, if a given belief base is supported by a large number of agents or sources involved in the fusion, then this belief base will be in the result of the fusion. Majority fusion operators make sense if all sources are considered to be independent. Egalitarist fusion modes behave differently since they try to minimize agent's individual dissatisfaction. In egalitarist approaches, which are for instance majority-independent merging operators, a repetition of a same piece of information has no impact on the result of fusion.

Both utilitarist and egalitarist approaches, when applied to merging ranked belief bases, are based on the assumption that belief bases to merge are commensurable. Namely all sources share a same common scale to order their information. This commensurability assumption may make sense in some applications, but can appear to be too strong for other applications.

This paper deals with egalitarist-based fusion modes, which are majority independent, of incommensurable ranked *DL-Lite* bases. We use the maximum-based merging operator as an example of egalitarist-based information fusion.

2 Possibilistic DL-Lite

In this section, we recall the main notions of possibilistic *DL-Lite* logic [1], denoted π-*DL-Lite*. This formalism is an extension of *DL-Lite* within the possibility theory setting. π-DL-$Lite$ provides a powerful and natural mechanism to deal with uncertainty and to ensure reasoning under inconsistency while keeping a computational complexity identical to the one used in standard *DL-Lite*.

2.1 DL-Lite logic

This section briefly recalls the main concepts of *DL-Lite* logic. For the sake of simplicity, we only consider $DL\text{-}Lite_R$ that underlies the *OWL2-QL* language. [4]. The $DL\text{-}Lite_R$ language is defined as follows:

$$\begin{array}{llll} B \longrightarrow A \mid \exists R & & C \longrightarrow B \mid \neg B \\ R \longrightarrow P \mid P^- & & E \longrightarrow R \mid \neg R \end{array}$$

where A is an atomic concept, P is an atomic role and P^- is the inverse of the atomic role. B (*resp.* C) is called basic (*resp.* complex) concept and role R (*resp.* E) is called basic (*resp.* complex) role. A *DL-Lite* KB is a pair $\mathcal{K}=\langle\mathcal{T},\mathcal{A}\rangle$ where $\mathcal{T}$ is called the TBox and $\mathcal{A}$ is called the ABox. A TBox includes a finite set of inclusion axioms on concepts and on roles respectively of the form: $B \sqsubseteq C$ and $R \sqsubseteq E$. The ABox contains a finite set of membership assertions on atomic concepts and on atomic roles respectively of the form $A(a)$ and $P(a,b)$ where a and b are two individuals. For the sake of simplicity, in the rest of this paper, when there is no ambiguity, we simply use *DL-Lite* instead of $DL\text{-}Lite_R$.

The semantics of a *DL-Lite* KB is given in term of interpretations. An interpretation $I=(\Delta^I,.^I)$ consists of an non-empty domain Δ^I and an interpretation function $.^I$ that maps each individual a to $a^I \in \Delta^I$, each A to $A^I \subseteq \Delta^I$ and each role P to $P^I \subseteq \Delta^I \times \Delta^I$. Furthermore, the interpretation function $.^I$ is extended in a straightforward way for complex concepts and roles for instance as follows: $(\neg B)^I=\Delta^I \setminus B^I$, $(P^-)^I=\{(y,x) \in \Delta^I \times \Delta^I | (x,y) \in P^I\}$ and

$(\exists R)^I = \{x \in \Delta^I | \exists y \in \Delta^I \, such\, that \, (x,y) \in R^I\}$. An interpretation I is said to be a model of a concept (*resp.* role) inclusion axiom, denoted by $I \models B \sqsubseteq C$ (*resp.* $I \models R \sqsubseteq E$), iff $B^I \subseteq C^I$ (*resp.* $R^I \subseteq E^I$). Similarly, we say that I satisfies a concept (*resp.* role) membership assertion, denoted by $I \models A(a)$ (*resp.* $I \models P(a,b)$), iff $a^I \in A^I$ (*resp.* $(a^I, b^I) \in P^I$).

A *DL-Lite* TBox $\mathcal{T}$ is said incoherent if there exists at least a concept C such that for each interpretation I which is a model of $\mathcal{T}$, we have $C^I = \emptyset$. A KB $\mathcal{K}$ is said consistent if it admits at least one model, otherwise $\mathcal{K}$ is said inconsistent. Note that within the *DL-Lite* setting, the inconsistency problem is always defined with respect to some *ABox* since a *TBox* may be incoherent but never inconsistent.

2.2 Possibility theory and *DL-Lite*

Let $\mathcal{L}$ be a *DL-Lite* description language, Ω be a universe of discourse consisting of a set of *DL-Lite* interpretations ($I = (\Delta, .^I) \in \Omega$). An epistemic state is represented by a possibility distribution π which is a mapping from Ω to the unit interval $[0,1]$ that assigns to each interpretation $I \in \Omega$ a possibility degree $\pi(I) \in [0,1]$. $\pi(I)$ represents the compatibility or consistency of I with respect to the set of available knowledge about the real world. When $\pi(I)=0$, I is said impossible and it is fully inconsistent with the set of available knowledge, whereas when $\pi(I)=1$, I is said totally possible and it is fully consistent with the available knowledge (namely nothing prevents I from being the real world). For two interpretations I and I', when $\pi(I) > \pi(I')$ we say that I is more consistent or more preferred than I' w.r.t the available knowledge. Lastly, π is said normalized if there exists at least one totally possible interpretation, namely $\exists I \in \Omega$, $\pi(I)=1$, otherwise, we say that π is sub-normalized. Note that the concept of sub-normalization reflects the presence of conflicts in the set of available information. Given a possibility distribution π defined on a set of interpretations Ω, two dual measures are generally used to assess the uncertainty of any event of interest $\phi \subseteq \Omega$: the possibility Π and the necessity N measures such that $\Pi(\phi) = \max_{I \in \phi}(\pi(I))$ and $N(\phi)$=1-$\Pi(\neg\phi)$. These two measures are extended for a *DL-Lite* axiom ϕ as follows:

- **Possibility measure:** $\Pi(\phi) = \max_{I \in \Omega}\{\pi(I) : I \vDash \phi\}$ evaluates to what extent an axiom ϕ is compatible with the available knowledge encoded by π.

- **Necessity measure:** The necessity degree $N(\phi) = 1 - \max_{I \in \Omega}\{\pi(I) : I \nvDash \phi\}$ evaluates to what extent ϕ is certainty entailed from the available knowledge encoded by π where $I \nvDash \phi$ means that I is not a model of ϕ.

2.3 π-*DL-Lite* knowledge bases

Syntactic representation Let $\mathcal{L}$ be a *DL-Lite* description language, a π-*DL-Lite* KB is a set of possibilistic axioms of the form (ϕ, α) where ϕ is an axiom expressed in $\mathcal{L}$ and $\alpha \in]0,1]$ is the degree of certainty of ϕ. Formally, $\mathcal{K}$=$\{(\phi_i, \alpha_i)$: i=1...$n\}$. Only somewhat certain information (α>0) is explicitly represented in a π-*DL-Lite* KB. A weighted axiom (ϕ, α) means that the certainty degree of ϕ is at least equal to α (namely, $N(\phi){\geq}\alpha$). A π-*DL-Lite* KB $\mathcal{K}$ will also be represented by a couple $\mathcal{K}$=$\langle\mathcal{T},\mathcal{A}\rangle$ where both elements in $\mathcal{T}$ and $\mathcal{A}$ may be uncertain. It is important to note that if for every axiom ϕ_i, we have α_i=1 then this gives a classical *DL-Lite* KB denoted $\mathcal{K}^*$=$\{\phi_i : (\phi_i, \alpha_i) \in \mathcal{K}\}$.

Given $\mathcal{K}$=$\langle\mathcal{T},\mathcal{A}\rangle$ a π-*DL-Lite* KB, we define the α-cut of $\mathcal{K}$ (*resp.* $\mathcal{T}$ and $\mathcal{A}$), denoted by $\mathcal{K}_{\geq\alpha}$ (*resp.* $\mathcal{T}_{\geq\alpha}$, $\mathcal{A}_{\geq\alpha}$), the sub-base of $\mathcal{K}$ (*resp.* $\mathcal{T}$ and $\mathcal{A}$) composed of axioms having weights α_i that are at least equal to α and the strict α-cut of $\mathcal{K}$ (*resp.* $\mathcal{T}$ and $\mathcal{A}$), denoted by $\mathcal{K}_{>\alpha}$ (resp. $\mathcal{T}_{>\alpha}$, $\mathcal{A}_{>\alpha}$), as a sub-base of $\mathcal{K}$ (*resp.* $\mathcal{T}$ and $\mathcal{A}$) composed of axioms having weights α_i strictly greater than α. We say that $\mathcal{K}$ is consistent if the standard base obtained from $\mathcal{K}$ by ignoring the weights associated with axioms is consistent. In case of inconsistency, we associate to $\mathcal{K}$ an inconsistency degree defined as follows:

Definition 1 *The inconsistency degree of a π-DL-Lite KB $\mathcal{K}$, denoted $Inc(\mathcal{K})$, is syntactically defined as follows: $Inc(\mathcal{K})$=$\max\{\alpha$:$\mathcal{K}_{\geq\alpha}$ is inconsistent$\}$.*

Semantics Given a π-*DL-Lite* KB $\mathcal{K}$, one can associate to $\mathcal{K}$ a joint possibility distribution, denoted by $\pi_{\mathcal{K}}$, defined over the set of all interpretations I=$(\Delta, .^I)$ by associating to each interpretation I its level of consistency with the set of available knowledge encoded in $\mathcal{K}$.

Definition 2 *The possibility distribution $\pi_{\mathcal{K}}$ induced from a π-DL-Lite KB $\mathcal{K}$ is defined as follows: $\forall I{\in}\Omega$:*

$$\pi_{\mathcal{K}}(I)=\begin{cases}1 \;\; if\, \forall\; (\phi_i, \alpha_i) \in \mathcal{K}, I \vDash \phi_i \\ 1 - max\{\alpha_i : (\phi_i, \alpha_i) \in \mathcal{K}, I \nvDash \phi_i\}\; otherwise\end{cases}$$

A π-*DL-Lite* KB $\mathcal{K}$ is said consistent if its joint possibility distribution $\pi_{\mathcal{K}}$ is normalized, otherwise $\mathcal{K}$ is said inconsistent and its inconsistency degree is defined semantically as follows:

Definition 3 *The inconsistency degree of a π-DL-Lite KB $\mathcal{K}$, denoted $Inc(\mathcal{K})$, is semantically defined as follows: $Inc(\mathcal{K})$=$1 - \max_{I\in\Omega}\{\pi_{\mathcal{K}}(I)\}$.*

Example 4 *Let $\mathcal{K}$=$\langle\mathcal{T},\mathcal{A}\rangle$ be a π-DL-Lite KB where $\mathcal{T}$=$\{(A{\sqsubseteq}B,.4)\}$ and $\mathcal{A}$=$\{(A(a),.5),(C(a),.7)\})$. One can compute $\pi_{\mathcal{K}}$ the possibility distribution induced from $\mathcal{K}$ using Definition 2.*
One can observe that $\pi_{\mathcal{K}}(I_8)$=1 meaning that the KB is consistent. Note that we have chosen a simple example in order to enumerate all interpretations. This will be helpful to illustrate the conditioning of a π-DL-Lite possibility distribution.

I	$.^I$	$\pi_{\mathcal{K}}$	I	$.^I$	$\pi_{\mathcal{K}}$
I_1	$A=\{\}$,$B=\{\}$,$C=\{\}$	.3	I_2	$A=\{a\}$,$B=\{\}$,$C=\{\}$	.3
I_3	$A=\{\}$,$B=\{a\}$,$C=\{\}$	.3	I_4	$A=\{\}$,$B=\{\}$,$C=\{a\}$	.5
I_5	$A=\{a\}$,$B=\{a\}$,$C=\{\}$	.3	I_6	$A=\{a\}$,$B=\{\}$,$C=\{a\}$	.6
I_7	$A=\{\}$,$B=\{a\}$,$C=\{a\}$	.5	I_8	$A=\{a\}$,$B=\{a\}$,$C=\{a\}$	1

Table 1: Example of a possibility distribution $\pi_{\mathcal{K}}$ computed using Definition 2.

2.4 Negative Possibilistic Closure in π-*DL-Lite*

Let $\mathcal{K}$=$\langle\mathcal{T},\mathcal{A}\rangle$ be a π-*DL-Lite* KB. In [1], it was shown that computing the inconsistency degree of $\mathcal{K}$ comes down to compute the one of $\langle\pi - neg(\mathcal{T}),\mathcal{A}\rangle$ where π-$neg(\mathcal{T})$ is the negated closure of $\mathcal{T}$. The negated closure will contain all the possibilistic negated axioms of the form ($\mathcal{K}_1$⊑¬$\mathcal{K}_2$,α) and (R_1⊑¬R_2,α) that can be derived from $\mathcal{T}$. The set π-$neg(\mathcal{T})$ is obtained by applying a set of rules that extend the ones defined in standard *DL-Lite* when axioms are weighted with certainty degrees. This notion is crucial for characterizing the concepts of consistency and how to deal with it in π-*DL-Lite* KB. At the beginning π-$neg(\mathcal{T})$ is set to an empty set.

Rule1: Add all negated axioms of $\mathcal{T}$ to π-$neg(\mathcal{T})$.

Rule2: If $\langle\mathcal{K}_1\sqsubseteq\mathcal{K}_2,\alpha_1\rangle\in\mathcal{T}$ and $\langle\mathcal{K}_2\sqsubseteq\neg\mathcal{K}_3,\alpha_2\rangle\in\pi$-$neg(\mathcal{T})$ then add $\langle\mathcal{K}_1\sqsubseteq\neg\mathcal{K}_3, min(\alpha_1,\alpha_2)\rangle$ to π-$neg(\mathcal{T})$.

Rule3: If $\langle\mathcal{K}_1\sqsubseteq\mathcal{K}_2,\alpha_1\rangle\in\mathcal{T}$ and $\langle\mathcal{K}_3\sqsubseteq\neg\mathcal{K}_2,\alpha_2\rangle\in\pi$-$neg(\mathcal{T})$ then add $\langle\mathcal{K}_1\sqsubseteq\neg\mathcal{K}_3, min(\alpha_1,\alpha_2)\rangle$ to π-$neg(\mathcal{T})$.

Rule4: If $\langle R_1\sqsubseteq R_2,\alpha_1\rangle\in\mathcal{T}$ and $\langle\exists R_2\sqsubseteq\neg B,\alpha_2\rangle\in\pi$-$neg(\mathcal{T})$ or $\langle B\sqsubseteq\neg\exists R_2,\alpha_2\rangle\in\pi$-$neg(\mathcal{T})$ then add $\langle\exists R_1\sqsubseteq\neg B,min\ (\alpha_1,\alpha_2)\rangle$ to π-$neg(\mathcal{T})$.

Rule5: If $\langle R_1\sqsubseteq R_2,\alpha_1\rangle\in\mathcal{T}$ and $\langle\exists R_2^-\sqsubseteq\neg B,\alpha_2\rangle\in\pi$-$neg(\mathcal{T})$ or $\langle B\sqsubseteq\neg\exists R_2^-,\alpha_2\rangle\in\pi$-$neg(\mathcal{T})$ then add $\langle\exists R_1^-\sqsubseteq\neg B,min\ (\alpha_1,\alpha_2)\rangle$ to π-$neg(\mathcal{T})$.

Rule6: If $\langle R_1\sqsubseteq R_2,\alpha_1\rangle\in\mathcal{T}$ and $\langle R_2\sqsubseteq\neg R_3,\alpha_2\rangle\in\pi$-$neg(\mathcal{T})$ or $\langle R_3\sqsubseteq\neg R_2,\alpha_2\rangle\in\pi$-$neg(\mathcal{T})$ then add $\langle R_1\sqsubseteq\neg R_3,min\ (\alpha_1,\alpha_2)\rangle$ to π-$neg(\mathcal{T})$.

Rule7: If $\langle R\sqsubseteq\neg R,\alpha\rangle\in\pi$-$neg(\mathcal{T})$ or $\langle\exists R\sqsubseteq\neg\exists R,\alpha\rangle\in\pi$-$neg(\mathcal{T})$ or $\langle\exists R^-\sqsubseteq\neg\exists R^-,\alpha\rangle\in\pi$-$neg(\mathcal{T})$ then add $\langle R\sqsubseteq\neg R,\alpha\rangle$ and $\langle\exists R\sqsubseteq\neg\exists R,\alpha\rangle$ and $\langle\exists R^-\sqsubseteq\neg\exists R^-,\alpha\rangle$ to π-$neg(\mathcal{T})$.

Rule8: $\langle\varphi,\alpha_1\rangle\in\pi$-$neg(\mathcal{T})$ and $\langle\varphi,\alpha_2\rangle\in\pi$-$neg(\mathcal{T})$ then $\langle\varphi,max(\alpha_1,\alpha_2)\rangle\in\pi$-$neg(\mathcal{T})$ where φ is an axiom of TBox or ABox.

The first rule simply states that negative axioms that are explicitly stated in $\mathcal{K}$ are trivially entailed from $\mathcal{K}$, and hence can be added to π-$neg(\mathcal{T})$. The rules 2-7 simply express transitivity relation induced by the inclusion relations. Rules 8 deals with redundancy and simply states that an axiom does not need to appear several times in a KB. It is enough to keep the one having the highest degree. Note that the minimum operation used in the rules for propagating certainty degrees is justified by the fact that the joint distribution will not be affected if the derived inclusion relations are added to the KB. Lastly, when the degrees α_i are equal to 1, then π-$neg(\mathcal{T})$ simply collapses with the standard negated closure defined for standard *DL-Lite* KBs. In fact, π-$neg(\mathcal{T})$ extends standard *DL-Lite* when one only deals with fully certain pieces of information.

2.5 Computing Inconsistency Degree in π-*DL-Lite*

We now provide a characterization of the inconsistency degree of a π-*DL-Lite* knowledge base by only focusing on $\langle\pi\text{-}neg(\mathcal{T}),\mathcal{A}\rangle$. First recall that the ABox only contains positive membership assertions (facts). Hence, the ABox alone is always consistent. Similarly, the TBox π-$neg(\mathcal{T})$ alone (namely, when ABox=$\emptyset$) is also consistent. Indeed, it is easy to define an interpretation I which is a model of π-$neg(\mathcal{T})$. For each $\langle\mathcal{K}_1\sqsubseteq\neg\mathcal{K}_2,\alpha\rangle\in\pi\text{-}neg(\mathcal{T})$, we let $(\mathcal{K}_i)^I=\emptyset$ if $\mathcal{K}_i$ is a concept and $(R)^I=\emptyset$ if $\mathcal{K}_i$ is of the form $\exists R$ or $\exists R^-$ and R is a role. I is then trivially a model of π-$neg(\mathcal{T})$. Hence, pieces responsible of inconsistency should involve both elements from π-$neg(\mathcal{T})$ and $\mathcal{A}$.

Besides, an inconsistency problem is always defined with respect to some ABox assertions and a TBox axiom, since a TBox may be incoherent but never inconsistent. A conflict, denoted by $\mathcal{C}$, is clearly an inconsistent subset of information that involve one element from π-$neg(\mathcal{T})$ and two elements from $\mathcal{A}$ [1]. It is minimal (up to a particular case where $\mathcal{K}_1$=$\mathcal{K}_2$). Indeed, removing any element of a conflict restores consistency. A particular case is when $\mathcal{K}_1\sqsubseteq\neg\mathcal{K}_1$ belongs to π-$neg(\mathcal{T})$. This corresponds to the situation of an unsatisfiable concept. A conflict hence involves one negative axiom from π-$neg(\mathcal{T})$ and one or two assertions. The following definition introduces the concepts of the degree of a conflict.

Definition 5 *Let $\mathcal{C}$ be a conflict. The degree of conflict, denoted $Deg(\mathcal{C})$, is defined as: $Deg(\mathcal{C})=min(\alpha_1,\alpha_2,\alpha_3)$ where $(D_1\sqsubseteq\neg D_2,\alpha_1)\in\mathcal{C}$,$(X,\alpha_2)\in\mathcal{C}$ and $(Y,\alpha_3)\in\mathcal{C}$ with X (resp. Y) is a concept or role assertion according to the form of D_1 (resp. D_2).*

The inconsistency degree of $\mathcal{K}$ ($Inc(\mathcal{K})$) using conflicts and their degrees is defined as follows:

Definition 6 *Let $\mathcal{K}=\langle\mathcal{T},\mathcal{A}\rangle$ be a π-DL-Lite KB and π-$neg(\mathcal{T})$ be its negated closure. Then:*

$$Inc(\mathcal{K})=Inc(\langle\pi-neg(\mathcal{T}),\mathcal{A}\rangle)=max\{Deg(\mathcal{C}):\mathcal{C}\text{ is a conflict of }\langle\pi\text{-}neg(\mathcal{T}),\mathcal{A}\rangle\}$$

3 Maximum-based merging

Let $\mathcal{L}$ be a $finite$ *DL-Lite* language. We denote by Ω the set of interpretations of $\mathcal{L}$ and by I an element of Ω. Greek letters ϕ, ψ denote *DL-Lite* axiom. $Mod(\phi)$ represents the set of models of ϕ, namely $Mod(\phi)=\{I\in\Omega : I\models\phi\}$.

There exists different ways to induce an ordering on possible interpretations from a given ranked belief base. In this paper, we use the so-called best-out ordering, which is defined as: an interpretation I is preferred to another interpretation I', if and only if the highest belief falsified by I is less important than the highest belief falsified by I'. Hence, each interpretation is associated with the ranks of highest axiom that it falsifies. Interpretations which are models of $\mathcal{K}_i^*$ have a rank equal to 0 and are the preferred ones. More precisely:

Definition 7 (Ranking functions) *A ranking function $\kappa_{\mathcal{K}_i}$ associated with a ranked DL-Lite KB $\mathcal{K}_i$ is a function that maps each interpretation $I \in \Omega$ to a natural number $\kappa_{\mathcal{K}_i}(I)$ such that:*

$$\kappa_{\mathcal{K}_i}(I) = \begin{cases} 0 \textit{ if } \forall(\phi_{ij},\ R_{\mathcal{K}_i}(\phi_{ij})) \in \mathcal{K}_i,\ I \models \phi_{ij} \\ max\{R_{K_i}(\phi_{ij}) : I \not\models \phi_{ij},\ (\phi_{ij}, R_{K_i}(\phi_{ij})) \in K_i\} \textit{ otherwise.} \end{cases}$$

The degree $\kappa_{\mathcal{K}_i}(I)$ will be called degree of surprise (or consistency degree) following Spohn's terminology [9]. Best-out ordering is the basis of possibilistic logic semantics [6] and adjustment revision [10].

3.1 Egalitarist-based fusion

Let $E = \{\mathcal{K}_1, ..., \mathcal{K}_n\}$ be a multi-set of n ranked bases issued from n sources, and let μ be a set of *DL-Lite* axioms representing integrity constraints to be satisfied. We suppose in this section that all the sources share the same meaning of ranks assigned to formulas. We also suppose that each ranked belief base is consistent (but of course, their union can be inconsistent).

The aim of merging is, given n commensurable ranked bases, to compute $\Delta(E)$, a *DL-Lite* KB representing the result of the fusion of these bases. In the literature, different methods for merging E have been proposed.

This paper focuses on an egalitarist fusion, and uses the maximum operator to illustrate the fusion process.

But first, we need to introduce the notion of *profile* associated with an interpretation I, denoted by $\nu_E(I)$, and defined by

$$\nu_E(I) =< \kappa_{\mathcal{K}_1}(I), ..., \kappa_{\mathcal{K}_n}(I) > .$$

It represents the degree of surprise (or dissatisfaction) of an interpretation I with respect to the multi-set of ranked bases.

The computation of the result of merging $\Delta(E)$ is done in two step: the surprise degrees $\kappa_{\mathcal{K}_i}(I)$'s are first combined with a merging operator (here the maximum operator), and the interpretations with lowest ranks are selected. This leads to define a strict order, denoted by $\lhd_{Max}$, between interpretations as follows: an interpretation I is preferred to another interpretation I' if the maximum element of the profile of I is smaller than the maximum element of the profile of I'. More formally:

Definition 8 (definition of $\lhd_{Max}$) *Let E be a multi-set of ranked DL-Lite bases. Let I and I' be two interpretations and $\nu_E(I)$, $\nu_E(I')$ be their associated profiles. Then:*

$$I \lhd^E_{Max} I' \text{ iff } Max(\nu_E(I)) < Max(\nu_E(I'))$$

where

$$Max(\nu_E(I)) = Max\{\kappa_{\mathcal{K}_i}(I) : i \in \{1, ..., n\}\}.$$

The result of the merging $\Delta^{max}_\mu(E)$ is a *DL-Lite* KB whose models are interpretations which are models of a constraint μ and which are minimal with respect to $\lhd_{Max}$. More formally:

Definition 9 (Maximum-based merging operator) *Let $E = \{\mathcal{K}_1, ..., \mathcal{K}_n\}$ be a multi-set of ranked DL-Lite belief bases and μ be an integrity constraint. The result of merging is a DL-Lite KB, denoted by $\Delta^{max}_\mu(E)$, defined by:*

$$Mod(\Delta^{max}_\mu(E)) = \{I \in Mod(\mu) : \nexists I' \in Mod(\mu), I' \lhd^E_{Max} I\}$$

4 Fusion-based on compatible scalings

The merging operation defined above assume that the sources, who provide $\mathcal{K}_i$'s, are commensurable.

Such an assumption is not always true. In the following, we drop this commensurability assumption.

We present in this section a strategy for an egalitarist fusion of incommensurable ranked *DL-Lite* belief bases. A natural way to make them commensurable is to apply a compatible scaling on existing ranks. A scaling is said to be compatible if it preserves original relative orders between beliefs of each ranked bases.

A scaling $\mathcal{S}$ assigns new ranks to beliefs of each ranked bases from the multiset E. Namely:

Definition 10 (compatible scaling) *Let $E = \{\mathcal{K}_1, ..., \mathcal{K}_n\}$ where $\mathcal{K}_i = \{(\phi_{ij}, R_{\mathcal{K}_i}(\phi_{ij}))\}$. Then a scaling $\mathcal{S}$ is defined by:*

$$\begin{array}{rccc} \mathcal{S}: & \mathcal{K}_1 \sqcup ... \sqcup \mathcal{K}_n & \rightarrow & \mathbb{N} \\ & (\phi_{ij}, R_{\mathcal{K}_i}(\phi_{ij})) & \mapsto & \mathcal{S}(\phi_{ij}) \end{array}$$

where $\sqcup$ represents union of multi-sets.

A scaling $\mathcal{S}$ is said to be compatible with $R_{\mathcal{K}_1}, ..., R_{\mathcal{K}_n}$ if and only if:

$$\forall \mathcal{K}_i \in E,\ \forall (\phi, R_{\mathcal{K}_i}(\phi)),\ (\phi', R_{\mathcal{K}_i}(\phi')) \in \mathcal{K}_i,$$

$$R_{\mathcal{K}_i}(\phi) \le R_{\mathcal{K}_i}(\phi') \ \text{ iff } \ \mathcal{S}(\phi) \le \mathcal{S}(\phi').$$

Clearly, a compatible scaling is not unique.

The set of compatible scalings with E is denoted by $\mathbb{S}_E$. Note that $\mathbb{S}_E$ is never empty. A straightforward compatible scaling is the one simply obtained by letting $\mathcal{S}(\phi_{ij}) = R_{\mathcal{K}_i}(\phi_{ij})$. It is compatible because it obviously preserves the relative ordering between beliefs of each base.

Given a compatible scaling $\mathcal{S}$, we denote by $\mathcal{K}_i^{\mathcal{S}}$ the ranked base obtained from $\mathcal{K}_i$ by replacing each pair $(\phi_{ij}, R_i(\phi_{ij}))$ by $(\phi_{ij}, \mathcal{S}(\phi_{ij}))$. Similarly, we denote by $E^{\mathcal{S}}$ the multi-set obtained from E by replacing each $\mathcal{K}_i$ in E by $\mathcal{K}_i^{\mathcal{S}}$.

A natural question now is, given the set of all compatible scalings $\mathbb{S}_E$, how to define the result of merging? Different options exist, either we use some uncertainty measure to select one compatible scaling from $\mathbb{S}_E$, or we consider all compatible scalings. In this paper, we adopt a skeptical option and consider all compatible scalings, in order to avoid arbitrary choices. An interpretation I is then said to be preferred to I', if for each compatible scaling $\mathcal{S}$, I is preferred to I' using Definition 8 (namely, $I \lhd_{Max}^{E^{\mathcal{S}}} I'$). More precisely,

Definition 11 (Ordering between interpretations) *Let E be a multi-set of ranked belief bases, $\mathbb{S}_E$ be the set of all compatible scalings associated with E. Let I, I' be two interpretations. Then:*

$$I <_{\forall}^{E} I' \quad \textit{iff} \quad \forall \mathcal{S} \in \mathbb{S}_E,\ I \lhd_{Max}^{E^{\mathcal{S}}} I'$$

where $\lhd_{Max}^{E^{\mathcal{S}}}$ is the result of applying Definition 8 on $E^{\mathcal{S}}$.

Models of $\Delta_{\mu}^{\forall}(E)$ are those which are models of μ and minimal for $<_{\forall}^{\mathcal{S}}$, namely:

$$Mod(\Delta_{\mu}^{\forall}(E)) = \{I \in Mod(\mu) :\ \nexists I' \in Mod(\mu),\ I' <_{\forall}^{E} I\}.$$

Note that $<_{\forall}^{E}$ is only a partial order.

The following proposition shows that an interpretation I is a model of $\Delta_{\mu}^{\forall}(E)$ if and only if there exists a compatible scaling where this interpretation belongs to the result fusion, namely it is a model of $\Delta_{Max}^{E^{\mathcal{S}}}(E)$. More formally:

Proposition 12 *Let E be a multi-set of ranked belief bases. Then $I \in Mod(\Delta_{\mu}^{\forall}(E))$, if and only if there exists a compatible scaling $\mathcal{S}$ such that $I \in Mod(\Delta_{\mu}^{max}(E^{\mathcal{S}}))$.*

5 Conclusion

This paper has addressed an issue which is not widely considered in belief fusion. It concerns the problem of merging incommensurable ordered belief bases. We have proposed a natural definition based on the idea of compatible rankings.

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