Redundancy in data fusion
ASPIQ research report

1 Introduction

In the last years, there has been a growing use of ontologies in many application areas where Description Logics (DLs for short) have been recognized as the powerful mechanism for formally representing and reasoning on ontologies[3]. An ontological DLs knowledge base is built up from two distinct component called TBox $\mathcal{T}$ and ABox $\mathcal{A}$. These two components represent respectively terminological (i.e. intensional, general) knowledges that describe application domain and assertional (i.e. extensional) knowledge that describes different facts (i.e. individuals). In recent web application, a scaling up of the size of ontologies has been identified. With the aim to ensure a tractable reasoning, various lightweight DLs have been recently proposed which offer important advantages with large terminological base (TBox) or assertional base (ABox). One of these DLs is *DL-Lite,* especially dedicated to applications that use huge volumes of data where query answering is the most important reasoning task. *DL-Lite* is a family of tractable DLs investigated by [11] where knowledge bases consistency check and all DLs standard reasoning services are polynomial for combined complexity (*i.e.* the overall size of the knowledge base) [2].

In several situations, the large volume of data may comes from several and not necessarily agreeing sources of information, for instance from web crawling. This problem is closely related to knowledge bases fusion or belief fusion (*e.g.* [10][15][16]) largely studied when knowledge bases are encoded in propositional logic framework. Belief fusion focuses on aggregating pieces of information issued from distinct, and may be conflicting or inconsistent, sources of information (*i.e.* agents). It leads to a global point of view of considered problems by taking advantage of pieces of information provided by each source. Generally, pieces of information provided by each source are represented, on one hand, syntacticly using a logical framework through a set of formulas, and on the other hand, semantically through a set of interpretations. Thus, two lines of approaches to merge pieces of information can be considered. Semantics approaches based on interpretations consist first to rank interpretations using some distance, then to combine them using an aggregation function to generally obtain a unique ordering on interpretations for all sources of information (see for instance [23] for more details). Syntactic approaches based on formulas consist to merge formulas of each knowledge base to obtain a unique knowledge base that represents all sources of information. Roughly speaking, merging pieces of information requires to perform some fusion operators that permit to combine them while respecting different constraints between sources. In the literature, several fusion operators have been proposed which depend on the nature and the representation of knowledge such as merging propositional knowledge bases (*e.g.* [23]), prioritized knowledge bases (*e.g.* [14]) or weighted logical knowledge bases (*e.g.* [7]).

When merging pieces of information issued from several sources a critically important problem arise. It consists on dealing with redundant information. Generally there exists two attitudes face to redundancy in information fusion. The first one consists on considering redundant information as a means of strengthening or confirmation especially when sources are independent. The second one consists on considering the redundancy as a repetition that can be ignored, especially when sources are dependent.

With the same sprit than belief fusion, recent works have been devoted to belief revision when knowledge bases are encoded within *DL-Lite* frameworks (e.g. [17, 18, 13, 27, 21]). In [13, 21] there has been point out that models-based approaches for *DL-Lite* knowledge base revision suffer from many problems and especially from the inexpressibly of the result of revision in the original fragment in which the knowledge base was encoded. However formulas-based approaches have given good result for *DL-Lite* knowledge bases revision (e.g. [13, 19, 6]).

In [23], it was shown that some formulas-based merging operators rather seems to the approaches dedicated to inconsistency-tolerance reasoning from an inconsistent belief based to merging operators. Recently several works (e.g. [20, 24, 8, 9]) have been proposed to perform inconsistency-tolerance inferences from inconsistent *DL-Lite* knowledge bases. These works are mainly inspired from the approaches proposed in databases which are based on the notion of repair to answering queries raised to inconsistent databases. A repair of a database contradicting a set of integrity constraints is a database obtained by applying a minimal set of changes in order to restore consistency. The notion of database repair has been extended to ABox repair for DL knowledge bases.

In this technical report, we study assertional-based fusion which comes down to aggregate several assertional bases, i.e, sets of data issued from distinct sources of information while resolving different conflicts between sources in order to ensure the consistency of the resulting *DL-Lite* knowledge base. This operation results from the effect of finding maximal assertional-based reparation. An important problem addressed here is how to deal with redundant data. Indeed, we go one step further on the definition of assertional-based reparation by tacking into account redundancy of data. Note that in this report, we study assertional-based reparation using both set inclusion criterion and cardinality criterion.

The rest of this report is organized as follows: Section 1 briefly recalls *DL-Lite* logics. Section 2 gives preliminaries on belief fusion in propositional logical setting. We first present rational postulates that logically characterize behaviors of merging operators, and then, we provide an overview of the main merging operators and their attitude face to redundancy. Section 4 investigates assertional-based fusion within *DL-Lite* framework and studies the impact of redundancy in assertional-based reparations. Section 5 concludes this report.

2 DL-Lite logic

The vocabulary of DLs is based on concepts which correspond to unary predicates to denote sets of individuals, and roles, which correspond to binary predicates, to denote binary relations among individuals. A description language is characterized by a set of constructs used to build complex concepts and roles form atomic ones and it is employed to structure a domain of interest. Each description language allows different sets of constructs. A DLs knowledge base is specified through several inclusion relations between concepts and roles.

In this paper, we focus on *DL-Lite* one of the most important fragment of DLs. For sake of simplicity, we only consider the language of $DL\text{-}Lite_{core}$ considered as the core language for all DLs in *DL-Lite* family. It is ensured by a description language defined as follow [12]:

$$B \longrightarrow A|\exists R \qquad R \longrightarrow P|P^{-} \qquad C \longrightarrow B|\neg B$$

where A is an atomic concept and P is an atomic role. B (*resp.* C) are called basic (*resp.* complex) concepts and roles R are called basic roles.

A *DL-Lite* knowledge base is a pair $\mathcal{K} = \langle \mathcal{T}, \mathcal{A} \rangle$ where $\mathcal{T}$ is a TBox and $\mathcal{A}$ is an ABox. The $DL\text{-}Lite_{core}$ TBox is constituted by a finite set of *inclusion assertions* of the form

$$B \sqsubseteq C,$$

Where B and C are concepts of the language defined above. Note that negation is not allowed in the left side of the inclusion assertion, while it can appear in the right side of an inclusion assertion.

Let us use a_i and a_j to denote two individuals (constants), the $DL\text{-}Lite_{core}$ ABox is constituted by a finite set of *membership assertions* on atomic concepts and on atomic roles of the form

$$A\left(a_i\right) \qquad \text{and} \qquad P\left(a_i, a_j\right)$$

For a more detailed description on *DL-Lite* family, see [2].

Note that $DL\text{-}Lite_{core}$ language does not allow the use of the conjunctive and the disjunctive operators. However, one can easily add conjunctions (*resp.* disjunctions) in the right-hand side (*resp.* left-hand side) of inclusion assertions. Indeed, as we will see it later, the conjunction of the form $B \sqsubseteq C_1 \sqcap C_2$ is equivalent to the pair of inclusion assertions $B \sqsubseteq C_1$ and $B \sqsubseteq C_2$, while the disjunction of the form $B_1 \sqcup B_2 \sqsubseteq C$ is equivalent to the pair of inclusion assertions $B_1 \sqsubseteq C$ and $B_2 \sqsubseteq C$.

As usual in DLs, the *DL-Lite* semantics is given by an interpretation $I = \left(\Delta, .^{I}\right)$ which consists of a non-empty domain Δ and an interpretation function

$.^I$. The function $.^I$ assigns to each individual a an element a^I of Δ^I, to each concept C a subset C^I of Δ^I and to each role R a binary relation R^I over Δ^I. Furthermore, the interpretation function $.^I$ is extended for the constructs of $DL\text{-}Lite_{core}$ as follow:

$$\begin{array}{rcl} A^I & \subseteq & \Delta^I \\ (P)^I & \subseteq & \Delta^I \times \Delta^I \\ \left(P^-\right)^I & = & \left\{(y,x) \in \Delta^I \times \Delta^I \mid (x,y) \in P^I\right\} \\ (\exists R)^I & = & \left\{x \in \Delta^I \mid \exists y \in \Delta^I \; such\, that \; (x,y) \in R^I\right\} \\ (\neg B)^I & = & \Delta^I \setminus B^I \\ (\neg R)^I & = & \Delta^I \times \Delta^I \setminus R^I \end{array}$$

An interpretation I is said to be a model of an inclusion assertion $B \sqsubseteq C$, denoted by $I \vDash B \sqsubseteq C$, if and only if $B^I \subseteq C^I$. Similarly, we say that an interpretation I is a model of membership assertion $A(a_i)$ (*resp.* $P(a_i, a_j)$), denoted by $I \vDash A(a_i)$ (*resp.* $I \vDash P(a_i, a_j)$), if and only if $a_i^I \in A^I$ (*resp.* $\left(a_i^I, a_j^I\right) \in P^I$). Note that we only consider $DL\text{-}Lite_{core}$ with unique name assumption (*i.e.* $a_i \neq a_j$ where $i \neq j$).

Thus, I is a model of knowledge base $\mathcal{K} = \langle \mathcal{T}, \mathcal{A} \rangle$, denoted by $I \vDash \mathcal{K}$, if and only if $I \vDash \mathcal{T}$ and $I \vDash \mathcal{A}$ where $I \vDash \mathcal{T}$ (*resp.* $I \vDash \mathcal{A}$) means that I is a model of all inclusion assertions in $\mathcal{T}$ (*resp.* membership assertions in $\mathcal{A}$). A knowledge base $\mathcal{K}$ is said to be consistent (or satisfiable) if it admits at least one model.

3 Belief fusion

In propositional logic framework, a belief base or a knowledge base, denoted by B_i, is constituted by a finite set of propositional formulas. Let use $\mathcal{B} = \{B_i, ..., B_n\}$ to denote a multi-set, called belief profile, to represent belief bases to be merged. Assume that each belief base is consistent. However merging multiple consistent belief bases may causes a conflict between bases. Hence, aggregation process requires to perform suitable merging operators in order to solve conflict.

Let use $\triangle$ to denote a merging operator and $\triangle(\mathcal{B})$ to denote the result of merging operation. In some case, a merging operator is submitted to some integrity constraints, denoted by ρ, generally expressed with a set of propositional formulas. Indeed, a merging operation with integrity constraints, denoted by $\triangle_\rho(\mathcal{B})$, must retains the integrity constraints explicitly represented in the merging result and not simply implicitly consistent with the merging result. A logical characterization of integrity constraints merging operators has been proposed in [23] through a set of rational postulates extended from the one proposed for belief revision (AGM postulates [1]).

3.1 Belief fusion postulates

In this section, we recall first rational postulates proposed to characterize the ideal behavior of a fusion operator. We then present the two main classes of merging operators, namely arbitration and majority merging operators. Finally, we give the representation theorem introduced in [23] of merging operators.

(**IC0**) $\triangle_\rho(\mathcal{B}) \models \rho$

(**IC1**) If ρ is consistent, then $\triangle_\rho(\mathcal{B})$ is consistent

(**IC2**) If $\bigwedge \mathcal{B}$ is consistent with ρ, then $\triangle_\rho(\mathcal{B}) \equiv \bigwedge \mathcal{B} \wedge \rho$

(**IC3**) If $\mathcal{B}_1 \equiv \mathcal{B}_2$ and $\rho_1 \equiv \rho_2$, then $\triangle_{\rho_1}(\mathcal{B}_1) \equiv \triangle_{\rho_2}(\mathcal{B}_2)$

(**IC4**) If $B \models \rho$ and $B' \models \rho$, then $\triangle_\rho(B \sqcup B') \wedge B \nvdash \bot \Rightarrow \triangle_\rho(B \sqcup B') \wedge B' \nvdash \bot$

(**IC5**) $\triangle_\rho(\mathcal{B}_1) \wedge \triangle_\rho(\mathcal{B}_2) \models \triangle_\rho(\mathcal{B}_1 \sqcup \mathcal{B}_2)$

(**IC6**) If $\triangle_\rho(\mathcal{B}_1) \wedge \triangle_\rho(\mathcal{B}_2)$ is consistent, then $\triangle_\rho(\mathcal{B}_1 \sqcup \mathcal{B}_2) \models \triangle_\rho(\mathcal{B}_1) \wedge \triangle_\rho(\mathcal{B}_2)$

(**IC7**) $\triangle_{\rho_1}(\mathcal{B}) \wedge \rho_2 \models \triangle_{\rho_1 \wedge \rho_2}(\mathcal{B})$

(**IC8**) if $\triangle_{\rho_1}(\mathcal{B}) \wedge \rho_2$ is consistent, then $\triangle_{\rho_1 \wedge \rho_2}(\mathcal{B}) \models \triangle_{\rho_1}(\mathcal{B}) \wedge \rho_2$

Intuitively, these postulates seek for a set of propositional formulas that represents the fusion result in the most faithful way as possible to aggregate belief bases while respecting integrity constraints. Indeed, the first postulate (**IC0**) statues that the fusion result must satisfies the integrity constraints. (**IC1**) ensures that if the set of integrity constraints is consistent then the fusion result must be also consistent. (**IC2**) means that whatever is possible, the fusion result is simply the conjunction of the merged belief bases and the integrity constraints. (**IC3**) simply refers to syntax-irrelevancy principle and it states that if two belief profiles are equivalents and that two sets of integrity constraints are logically equivalent then the resulting belief base of each belief profile are logically equivalents. (**IC4**) assures that when merging a pairs of belief bases then the merging operator must not gives any preference to one of them. (**IC5**) and (**IC6**) together affirm that if possible one can find two subgroups of sources which are in agreement on at least one interpretation, then the fusion result is exactly the interpretations on which the two groups are in agreement. Finally, (**IC7**) and (**IC8**) are introduced to preserve the notion of closeness (early presented in AGM postulates [1] through R5 and R6 postulates).

To define the iteration property of a merging operator, one add the following alternative postulate.

(**IC**$_{it}$) If $B \models \rho$ then $\exists n, \triangle^n_\rho(\mathcal{B}, B) \models B$

This postulate stats that if a merging operator gives the average belief of a belief profiles, then if we take this fusion result and iterate it with the same belief base, we reach after a number of iteration this belief base.

In [23] merging operators have been classified into two majors subclasses, namely majority merging operators and arbitration merging operators. Formally, the family of majority merging operators is logically characterized by the following postulate:

(**Maj**) $\exists n$ such that $\triangle_\rho(\mathcal{B}_1 \sqcup \mathcal{B}_2^n) \models \triangle_\rho(\mathcal{B}_2)$

This postulate affirms that if a particular set of belief is repeated quite enough in the whole set of belief profile then this particular set of beliefs must be prevail in the fusion result. Hence, it is obvious that majority operators behaviors is sensitive to redundancy. Namely, a majority operator views the redundancy as a kind of confirmation.

Note that, face to redundancy another strategy can be performed. It requires that the fusion result must be independent from redundancy beliefs. Hence, one can define majority independence merging operators which they satisfy the following postulate:

(**MI**)$\exists n$ such that $\triangle_\rho(\mathcal{B}_1 \sqcup \mathcal{B}_2^n) \models \triangle_\rho(\mathcal{B}_1 \sqcup \mathcal{B}_2)$

As it was shown in [23], majority independence operators are not compatible with integrity constraints merging operators. Indeed, a weak form of (**MI**) postulate, called weak majority independence, was proposed. This postulate states that there exists some cases where the fusion result is independent of the majority of opinions and it is defined as follow:

(**WMI**) $\forall B', \exists B : B' \not\models B\ \exists n, \triangle_\rho(B' \sqcup B^n) \equiv \triangle_\rho(B' \sqcup B)$

Contrarily to majority merging operators that take into account the opinion of the majority about a situation, arbitration merging operators try to better satisfies as possibles opinions among integrity constraints [23]. Namely, if a set of preferred opinion among a set of integrity constraints ρ_1 for a belief base B correspond to the preferred opinion among the set of integrity constraints ρ_2 of another base B' and if the opinion that belong to a set of integrity constraints but not to the other are equally preferred for the whole groups $(B \sqcup B')$, then the subset of preferred opinions among the disjunction of integrity constraints will coincide with the preferred opinions of each base among their respective integrity constraints. Formally, the arbitration merging operators are logically characterized by the following postulate:

$$(\textbf{Arb}) \quad \left.\begin{array}{l} \triangle_{\rho_1}(B_1) \equiv \triangle_{\rho_2}(B_2) \\ \triangle_{\rho_1 \leftrightarrow \neg\rho_2}(\{B_1, B_2\})\,(\rho_1 \leftrightarrow \neg\rho_2) \\ \rho_1 \not\models \rho_2 \\ \rho_2 \not\models \rho_1 \end{array}\right\} \Rightarrow \triangle_{\rho_1 \vee \rho_2}(\{B_1, B_2\}) \equiv \triangle_{\rho_1}(B_1)$$

Lastly, we point out that when the fusion operator is not submitted to any integrity constraints, the merging operation is called pure merging and the postulates presented above are expressed as follow:

(**P0**) $\triangle(\mathcal{B})$ is consistent

(**P1**) if $\mathcal{B}$ is consistent, then $\triangle(\mathcal{B}) \equiv \bigwedge \mathcal{B}$

(**P2**) if $\mathcal{B}_1 \equiv \mathcal{B}_2$, then $\triangle(\mathcal{B}_1) \equiv \triangle(\mathcal{B}_2)$

(**P4**) if $B \wedge B'$ is inconsistent, then $\triangle(B \sqcup B') \wedge B \nvdash \bot$

(**P5**) $\triangle(\mathcal{B}_1) \wedge \triangle(\mathcal{B}_2) \models \triangle(\mathcal{B}_1 \sqcup \mathcal{B}_2)$

(**P6**) if $\triangle(\mathcal{B}_1) \wedge \triangle(\mathcal{B}_2)$ is consistent, then $\triangle(\mathcal{B}_1 \sqcup \mathcal{B}_2) \models \triangle(\mathcal{B}_1) \wedge \triangle(\mathcal{B}_2)$

Now, according to rational postulates **IC0-IC8**, an integrity constraints merging operator is logically defined by a representation theorem according to the notion of syncretic assignment [23]. The syncretic assignment try to built up a pre-orders (i.e. preference relations) on interpretations and it is defined in the same way as faithful assignment defined for beliefs revision [1].

Definition 1. (Syncretic assignment) Let ω and ω' be two interpretations. A syncretic assignment is a function mapping each belief set $\mathcal{B}$ to a total pre-orde $\leq_{\mathcal{B}}$ over interpretations such that for any belief sets $\mathcal{B}, \mathcal{B}_1, \mathcal{B}_2,$ and for any belief bases B, B' the following conditions hold:

1. if $\omega \models \mathcal{B}$ and $\omega' \models \mathcal{B}$, then $\omega \simeq_{\mathcal{B}} \omega'$
2. if $\omega \models \mathcal{B}$ and $\omega' \nvDash \mathcal{B}$, then $\omega <_{\mathcal{B}} \omega'$
3. if $\mathcal{B}_1 \equiv \mathcal{B}_2$ then $\leq_{\mathcal{B}_1} = \leq_{\mathcal{B}_2}$
4. $\forall \omega \models B$, $\exists \omega' \models B'$ such that $\omega' \leq_{\{B \sqcup B'\}} \omega$
5. if $\omega \leq_{\mathcal{B}_1} \omega'$ and if $\omega \leq_{\mathcal{B}_2} \omega'$, then $\omega' \leq_{\{\mathcal{B}_1 \sqcup \mathcal{B}_2\}} \omega$
6. if $\omega \leq_{\mathcal{B}_1} \omega'$ and if $\omega <_{\mathcal{B}_2} \omega'$, then $\omega' <_{\{\mathcal{B}_1 \sqcup \mathcal{B}_2\}} \omega$

The meaning of these conditions is the following: Conditions 1 and 2 state that the interpretations which are models of the belief profile are the more plausible for the pre-order associated to this belief profile. Condition 3 simply asserts that two equivalent belief profiles have the same joint pre-order. Condition 4 ensures that when two belief bases are merged, then there not exists any priorities between them. Condition 5 affirms that if an interpretation ω is preferred that an interpretation ω' for two belief profiles, then ω still preferred than ω' for the group composed by the two belief profiles. Finally, condition 6 strengthen condition 5 when ω is strictly preferred than ω'. Note that Condition 1-3 correspond to the one defined in faithful assignment in belief revision whereas Condition 5-6 correspond the Pareto conditions in social choice theory.

According to the notion of syncretic assignment, the representation theorem for integrity constraints merging operators is stated as follow:

Theorem 1. *An operator $\triangle$ is an integrity constraint merging operator if and only if there exists a syncretic assignment that maps each belief set $\mathcal{B}$ to a total pre-order $\leq_{\mathcal{B}}$ such that*

$$mod\left(\triangle_{\rho}\left(\mathcal{B}\right)\right) = min\left(mod\left(\rho\right), \leq_{\mathcal{B}}\right)$$

Hence, the result of merging the belief set with the merging operator $\triangle$ is simply represented by the pre-order $\leq_{\mathcal{B}}$ on the interpretations that consists on computing minimal interpretations to the pre-order associated to belief bases to be merged.

3.2 Example of merging operators

Generally, pieces of information provided by each source are represented, on one hand, syntacticly using a propositional set of formulas, and on the other hand, semantically through a set of interpretations. Thus, one can identify two categories of merging operators. Semantics merging operators based on interpretations, well known as models-based merging operators, that consist first to rank interpretations using some distance, then to combine them using an aggregation function to generally obtain a unique ordering on interpretations for all sources of information. Syntactic merging operators based on formulas, well-known as formulas-based merging operators, that consist on aggregating formulas of each belief base to obtain a unique consistent belief base that represents all sources of information. In what follow, we present an briefly overview of the main merging operators proposed in the literature for each categories.

3.2.1 Models-based merging operators

The models-based merging operators are defined on distance between interpretations. A distance, denoted by d, is mapping from $\Omega \times \Omega$ to $\mathbb{N}$ such that:

- $d\left(\omega, \omega'\right) = d\left(\omega', \omega\right)$, and
- if $\omega = \omega'$ then $d\left(\omega, \omega'\right) = 0$.

Hence, the distance between an interpretation and a belief base can be defined as follow:

$$d\left(\omega, B\right) = min\left\{d\left(\omega, \omega'\right) : \omega \models B\right\}$$

The early work addressing on model-based fusion was proposed in [26] by introduicing the so-called model fitting operators which are based on arbitration between two belief bases. However, the real works on belief bases aggregation was defined in [22] as integrity merging operators.

The $\sum$ merging operators

The Sum merging operators, denoted by $\sum$, are considered as majority merging operators.Thus, the distance between an interpretation ω and a belief profile $\mathcal{B}$ is defined as the sum of all distance between the interpretation and each belief bases B_i. Formally, let ω be an interpretation, then

$$d_{\Sigma}(\omega, \mathcal{B}) = \sum_{B_i \in \mathcal{B}} d(\omega, B_i)$$

Such definition gives a pre-order between interpretations as follow:

$$\text{if } d_{\Sigma}(\omega, \mathcal{B}) \leq d_{\Sigma}(\omega', \mathcal{B}) \text{ then } \omega \leq_{\mathcal{B}}^{\Sigma} \omega'$$

Hence the $\sum$ merging operators, denoted by $\triangle_{\Sigma}$ are defined by:

$$mod\left(\triangle_{\rho}^{\Sigma}(\mathcal{B})\right) = min\left(mod(\rho), \leq_{\mathcal{B}}^{\Sigma}\right)$$

The Max merging operators

The Max merging operators are considered also an approximation of arbitration merging operators. The distance between an interpretation ω and a belief profile $\mathcal{B}$ is defined as the max distance between an interpretation an the set of merged belief bases. Formally the max distance is defined as follow:

$$d_{Max}(\omega, \mathcal{B}) = \max_{B_i \in \mathcal{B}} d(\omega, B_i)$$

Thus, the pre-order between interpretations is defined as follow:

$$\text{if } d_{Max}(\omega, \mathcal{B}) \leq d_{Max}(\omega', \mathcal{B}) \text{ then } \omega \leq_{\mathcal{B}}^{Max} \omega'$$

By consequence, the Max merging operators, denoted by $\triangle_{Max}$ is defined by:

$$mod\left(\triangle_{\rho}^{Max}(\mathcal{B})\right) = min\left(mod(c), \leq_{\mathcal{B}}^{Max}\right)$$

The $GMax$ merging operators

The $GMax$ merging operators are simply a refinement of the Max merging operators with the aim to allow the $\triangle_{Max}$ operators to attend the arbitration behavior. Clearly $GMax$ family of merging operators are a lexicographical refinement of $GMax$ family of merging operators.

Let ω be an interpretation, we build the list $(d_1^{\omega}, ..., d_n^{\omega})$ of distances between this interpretation and the n belief bases in $\mathcal{B}$, i.e. $d_i^{\omega} = d(\omega, B_i)$. Let $L_{\omega}^{\mathcal{B}}$ be the list obtained from $(d_1^{\omega}, ..., d_n^{\omega})$ by sorting it in descending order. We will denote by $d_{GMax} = d(\omega, \mathcal{B})$ the list $L_{\omega}^{\mathcal{B}}$. Let $\leq_{lex}$ be the lexicographical order between sequences of integers (of the same length). We define the following total pre-order:

$$\omega \leq_{GMax}^{\mathcal{B}} \omega' \text{ if and only if } L_{\omega}^{\mathcal{B}} \leq_{lex} L_{\omega'}^{\mathcal{B}}$$

By consequence, the $GMax$ merging operators , denoted by $\triangle_{GMax}$ is defined by:

$$mod\left(\triangle_{\rho}^{GMax}(\mathcal{B})\right) = min\left(mod(c), \leq_{\mathcal{B}}^{GMax}\right)$$

3.2.2 Formulas-based merging operators

The formulas-based merging operators are based on the syntax of belief bases. Namely the fusion result which must be a consistent set of formulas depend to the syntactic represented of aggregated belief bases.

Combination operators

This family of merging operator, introduced by [4, 5], try to find from the unions of merged belief bases, consistent and maximal subsets of formulas. Note that the maximality criterion here is in the sense of set inclusion. Formally, let use $MAXCONS\,(B,\rho)$ to denote the collection of the subsets maximal consistent from $B \cup \rho$ which necessarily the integrity constraints ρ. Namely, a maximal consistent subset $M \in MAXCONS\,(B,\rho)$ satisfies the following requirements:

- $M \subseteq B \cup \rho$
- $\rho \in M$
- if $M \subset M' \subseteq \{B \cup \rho\}$ then M' is inconsistent

From now, let $MAXCONS(\mathcal{B},\rho){=}MAXCONS(\bigcup_{B_i \in \mathcal{B}} B_i,\rho)$ and when the maximality is in the sense of cardinality criterion, we will use $MAXCONS_{card}(\mathcal{B},\rho)$ as notation. Thus, one can define the combination operators as follow:

Definition 2. Let $\mathcal{B}$ a belief set an ρ an integrity constraint:

- $\triangle_{\rho}^{C_1}(\mathcal{B}) = \bigvee MAXCONS\,(\mathcal{B},\rho)$
- $\triangle_{\rho}^{C_3}(\mathcal{B}) = \bigvee\{M : M \in MAXCONS\,(\mathcal{B},\top)\ and\ M \cup \{\rho\}\ is\ consistent\}$
- $\triangle_{\rho}^{C_4}(\mathcal{B}) = \bigvee MAXCONS_{card}\,(\mathcal{B},\rho)$
- $\triangle_{\rho}^{C_5}(\mathcal{B}) = \bigvee\{M \cup \{\rho\} : M \in MAXCONS\,(\mathcal{B},\top)\ and\ M \cup \{\rho\}\ is\ consistent\}$ if this set is not empty otherwise ρ.

Intuitively, $\triangle_{\rho}^{C_3}(\mathcal{B})$, $\triangle_{\rho}^{C_4}(\mathcal{B})$ and $\triangle_{\rho}^{C_5}(\mathcal{B})$ are defined from $\triangle_{\rho}^{C_1}(\mathcal{B})$ and all those combination operators try to built up a consistent information form inconsistent one.

In [23], it was pointed out that combination operators rather seems to the approaches dedicated to inconsistency-tolerance reasoning from an inconsistent belief based to merging operators. Hence, combination operators does not exploited the repartition of the information between sources. Besides, they generally lost the original information provided by merged information sources. To overcome this inconvenient behaviors of combination operators, one can define selection factions in the same way as beliefs revision. Those section function try to select from the whole maximal consistent sets a group in order to attenuate as possible we can the fusion result produced by combination operators.

In [23] three selection functions was proposed. The first one, denoted by $\triangle^d$, selects the maxcons set which are consistent with most of merged beliefs bases.

The second one, denoted by $\triangle^{S,\Sigma}$, selects they maxcons which have the less symmetric difference in term of cardinality with the belief base. The last one, denoted by $\triangle^{\cap,\Sigma}$, selects maxcons which intersect at most with the bases.

DA^2 operators

The DA^2 operators is a family of merging operator based on a distance (D) and two aggregation function (A^2). Similarity defined as models-based ones, The DA^2 operators capture on one hand take into account the inconsistent of merged belief bases and on the other hand the distribution of information between information sources.

Definition 3. Let d be a distance between interpretations and f and g two aggregation functions. A DA^2 merging operator, denoted by $\triangle^{d,f,g}$ is defined as follow:

$$mod\left(\triangle_{\rho}^{d,f,g}(\mathcal{B})\right) = min\left(mod\left(\rho\right), \leq_{\mathcal{B}}\right)$$

where the pre-order $\leq_{\mathcal{B}}$ is defined as follow:
- $\omega \leq_{\mathcal{B}} \omega'$ if and only if $d(\omega, \mathcal{B}) \leq d(\omega', \mathcal{B})$
- $d(\omega, \mathcal{B}) = f(d(\omega, B_1), ..., d(\omega, B_n))$ where $\mathcal{B} = \{B_1, ..., B_n\}$
- $d(\omega, B_i) = g(d(\omega, \phi_1), ..., d(\omega, \phi_m))$ where $B_i = \{\alpha_1, ..., \alpha_m\}$.

4 Redundancy in assertional-based fusion

Now, we study redundancy in assertional-based fusion where knowledge bases are encoded within *DL-Lite* framework. Assertional-based fusion is closely related to ontology-based data access setting where a TBox is generally assumed to be a well formed and coherent ontology and the set of data, i.e, the set of membership assertions, is issued from several and not necessarily aggregating sources. In such setting each provided assertional base is assumed to be consistent with the ontology. However, the aggregation of all assertional bases may leads to an inconsistency problem. As mentioned in section 3, there exists different strategies to merging information where several fusion operators have been proposed. Generally, a fusion operator focus on aggregating information while solving every conflicts between sources and ensuring the consistency of result.

In [23], it was pointed out that combination operators, presented in the above section, rather seems to the approaches dedicated to inconsistency-tolerance reasoning from an inconsistent belief based to merging operators. Recently several works (e.g. [20, 24, 8, 9]) have been proposed to deal with inconsistency-tolerance inference in inconsistent *DL-Lite* knowledge bases framework. These works are especially inspired by the approaches proposed in databases which are based on the notion of database repair to answering queries raised to inconsistent databases. A repair of a database contradicting a set of integrity constraints is a database obtained by applying a minimal set of changes in order to restore consistency. The notion of database repair has been extended to ABox repair for *DL-Lite* knowledge bases. Let $\mathcal{K} = \langle \mathcal{T}, \mathcal{A} \rangle$ be a *DL-Lite* knowledge base, several notions of ABox repair have been provided. For instance in [20, 24] the ABox Repair (AR) is defined as a maximal subset of $\mathcal{A}$ w.r.t. set inclusion consistent with $\mathcal{T}$. Another way to define an assertional base reparation was proposed [6]. It consists on finding a maximal subset of $\mathcal{A}$ consistent with $\mathcal{T}$ based on cardinality criterion not set inclusion criterion.

In this section, we study assertional-based reparation taking into account the redundancy of assertional facts. We will focus on reparation as maximal set of assertional fact consistent with the ontology (TBox) using cardinality criterion and set inclusion criterion.

4.1 Redundancy representation

Let $\mathcal{L}$ be a *DL-Lite* language described above in Section 2 and $\mathcal{A}_1, ..., \mathcal{A}_k$ be k sets of assertional base provided by several sources linked to a terminological base $\mathcal{T}$. We assume that $\mathcal{T}$ is coherent and each assertional base is consistent with $\mathcal{T}$. We denote by $\mathcal{A}_\oplus$ a multi-set of membership assertions formed by aggregating all assertional bases which may be inconsistent with the TBox $\mathcal{T}$. Here we use $\mathcal{A}_\oplus$ as a multi-set since the same assertional fact, denoted by ϕ, may be present more than once. Namely, the same fact may be provided by distinct sources of information. Note that we assume that all sources that provide data have the same degree of reliability. Let $\mathcal{K} = \langle \mathcal{T}, \mathcal{A}_\oplus \rangle$ be a possibly *DL-Lite* knowledge base. In order to represent the redundancy of an assertional fact, we define the ABox $\mathcal{A}_\oplus$ as follow:

Definition 4. Let $\mathcal{K} = \langle \mathcal{T}, \mathcal{A}_\oplus \rangle$ be *DL-Lite* knowledge base where the ABox $\mathcal{A}_\oplus$ is a multi-set of membership assertions of the form (ϕ, n) such that:

- ϕ is a *DL-Lite* membership assertion.
- $n \in \mathbb{N}$ the number of occurrence of ϕ in $\mathcal{A}_1, ..., \mathcal{A}_k$.

Example 1. Let $\mathcal{T} = \{B \sqsubseteq A, C \sqsubseteq A, B \sqsubseteq \neg C, \exists s \sqsubseteq B, \exists s^- \sqsubseteq D, D \sqsubseteq \neg F\}$ be a terminological base and $\mathcal{A}_1 = \{D(b), s(a,b)\}$, $\mathcal{A}_2 = \{B(a), s(a,b)\}$, $\mathcal{A}_3 = \{C(a), D(b)\}$ be three ABox issued from different sources. Using Definition 4 the ABox $\mathcal{A}_\oplus$ is as follow: $\mathcal{A}_\oplus = \{(D(b), 2), (s(a,b), 2), (B(a), 1), (C(a), 1)\}$

When faced with redundant assertional facts, it is important to know if this redundancy brings a relevant information or not. Intuitively, one can see the redundancy as a means of confirmation. Namely, if we have two assertional facts (A, i) and(B, j) where $i < j$, then we consider (B, j) as more plausible than (A, i) since it was confirmed by more sources. Moreover, there exists another case where the redundancy is not relevant. Namely, when an assertional fact appears more than once but it can be deduced from other facts. Indeed, one can distinguish two types of redundancy, namely direct redundancy and indirect redundancy. More formally:

Definition 5. Let $\mathcal{K} = \langle \mathcal{T}, \mathcal{A} \rangle$ be a consistent *DL-Lite* knowledge base. Then:

- A membership assertion ϕ is indirectly redundant if $\mathcal{K} - \{\phi\} \models \phi$

- A membership assertion ϕ is directly redundant if $\mathcal{K} - \{\phi\} \not\models \phi$

Let use $ind(\mathcal{A}_i)$ to denote the set of indirectly redundant assertional facts of an ABox $\mathcal{A}_i$ relative to the TBox $\mathcal{T}$. Here the set $ind(\mathcal{A}_i)$ is a set, not a multi-set. We denote by $ind\,(\mathcal{A}_{\oplus}) = \bigcup ind\,(A_i)$ the set of indirectly redundant assertional facts of the assertional base $\mathcal{A}_{\oplus}$.

Example 2. Let us continue with Example 1. Using definition 5, one can check that $(D(b), 2)$ is an indirectly redundant assertional fact since $\mathcal{K} - \{(D(b)\} \models (D(b)$. However one can check that $s(a, b)$ is a directly redundant fact since $\mathcal{K} - \{s(a, b)\} \not\models s(a, b)$.

Although each assertional base $\mathcal{A}_k$ is assumed to be consistent with the TBox $\mathcal{T}$, their aggregation is not necessarily consistent with $\mathcal{T}$. Recall that in a *DL-Lite* setting, the inconsistency problem is always defined with respect to some ABox, since a TBox alone is never inconsistent. Hence, restoring consistency comes down to removing some conflict assertional facts with respect to the TBox. Let $\mathcal{K}$ be an inconsistent *DL-Lite* knowledge base, we define the notion of conflict which is a minimal inconsistent subset of $\mathcal{A}$, more formally:

Definition 6. Let $\mathcal{K} = \langle \mathcal{T}, \mathcal{A} \rangle$ be an inconsistent *DL-Lite* knowledge base. A conflict set C is a set of membership assertion such that:.

- $C \subseteq \mathcal{A}$,
- $\langle \mathcal{T}, C \rangle$ is inconsistent,
- $\forall C'$, $C' \subset C$, $\mathcal{T} \cup C'$ is consistent.

In the following, we use $C(\mathcal{K})$ to denote the collection of conflicts in $\mathcal{K}$. Note that $C(\mathcal{K})$ is a finite since $\mathcal{K}$ is assumed to be finite.

Example 3. Let us continue Example 1 where $\mathcal{K} = \langle \mathcal{T}, \mathcal{A}_{\oplus} \rangle$ is inconsistent. From $\mathcal{K}$, the set $C(\mathcal{K}) = \{\{(s(a, b), 2), (C(a), 1)\}, \{(B(a), 1), (C(a), 1)\}\}$.

Form example 3, one can easily observe that another form of redundancy may implicitly appears when computing conflict sets. It concerns occurrence of an assertional fact in the conflict sets. This new information may be helpful in restoring inconsistency. Namely, if there exists two assertional facts (A, i) and (B, j) involved in a conflict where $i < j$, then it is more plausible to throw out (B, j) then (A, i) in order to restore consistency.

Definition 7. Let $C(\mathcal{K})$ the collection of conflicts in $\mathcal{K}$, the number of occurrence of an assertional fact in conflict sets is defined as follow:

$$N\left((\phi, n)\right) = \sum (\phi, n) \in C\left(K\right)$$

Example 4. Continued with Example 3, one can check that $N\left((s(a, b), 2)\right) = 1$, $N\left(C(a), 1\right) = 2$ and $N\left((B(a), 1)\right) = 1$.

For sake of simplicity, we use in the following (ϕ, c) to denote the number of occurrence of an assertional fact ϕ in $C(\mathcal{K})$.

4.2 Redundancy in assertional-based reparation

In a *DL-Lite* setting, the notion of ABox reparation, which is closely related to the notion of databases reparation, consists on obtaining by applying a minimal set of changes a consistent ABox with respect to the TBox. In order to restore consistency, one can considering all maximal reparation. More precisely, a maximal assertional reparation is defined as follow:

Definition 8. Let $\mathcal{K} = \langle \mathcal{T}, \mathcal{A} \rangle$ be an inconsistent *DL-Lite* knowledge base. A maximal assertional reparation, denoted by $MAR(\mathcal{K})$, of $\mathcal{K}$ is a sub-set of $\mathcal{A}$ such that:

- $\mathcal{A}_{\mathcal{R}} \subseteq \mathcal{A}$
- $\exists \mathcal{A}_{\mathcal{R}'}$, $\mathcal{A}_{\mathcal{R}} \subsetneq \mathcal{A}_{\mathcal{R}'}$ and $\langle \mathcal{T}, \mathcal{A}_{\mathcal{R}'} \rangle$ is inconsistent.

Example 5. Continued with Example 1 and 3. Using Definition 3, one can obtain the following reparation: $R1 = \{(D(b), 2), (s(a, b), 2), (B(a), 1)\}$ and $R2 = \{(D(b), 2), (C(a), 1)\}$.

In order to restore consistency, there may exists several reparation where each of them leads to a possible consistent knowledge base. However, within the *DL-Lite* language it is not possible to find a knowledge base that represents all solution since we are not allowed to use a disjunctive operator ($\vee$). Traditionally, the maximal assertional reparation was proposed for flat ABox. However, when dealing redundancy represented through a weighted ABox several situation may hold depending on the nature of the redundancy.

According to weights associated to assertional facts in the ABox $\mathcal{A}_{\oplus}$, one can find a preference on their inclusion into the assertional-base reparations of $\mathcal{A}_{\oplus}$. More precisely, for two assertional facts the following proposition holds.

Proposition 1. *Let* (ϕ_i, n_i) and (ϕ_j, n_j) be two assertional facts of $\mathcal{A}_{\oplus}$ where $n_j > n_i$. *Then* $(\phi_i, n_i) \in MAR(\mathcal{K})$ and $(\phi_j, n_j) \notin MAR(\mathcal{K})$.

Clearly, the preference is given for assertional facts more confirmed by sources. Now, let $X \subseteq \mathcal{A}_{\oplus}$ be a set of $\mathcal{A}_{\oplus}$, we denote by:

$$max\,(X) = max\,\{n_i : (\phi_i, n_i) \in X \,and\, \phi_i \notin ind\,(\mathcal{A}_{\oplus})\}$$

the maximum weight associated to assertional facts of a set X, excepts indirectly redundant assertional facts (i.e, $\phi \in ind\,(\mathcal{A}_{\oplus})$).

By considering redundancy as means of confirmation, the following definition of assertional-based reparation holds.

Definition 9. Let $\mathcal{K} = \langle \mathcal{T}, \mathcal{A}_{\oplus} \rangle$ be an inconsistent *DL-Lite* knowledge base. A maximal assertional reparation of the multi-set $\mathcal{A}_{\oplus}$, denoted by $MAR(\mathcal{A}_{\oplus})$, is a sub-set of $\mathcal{A}_{\oplus}$ such that:

- $\mathcal{A}_{\mathcal{R}} \subseteq \mathcal{A}_{\oplus}$
- $\exists \mathcal{A}_{\mathcal{R}'}$, $\mathcal{A}_{\mathcal{R}} \subsetneq \mathcal{A}_{\mathcal{R}'}$ and $\langle \mathcal{T}, \mathcal{A}_{\mathcal{R}'} \rangle$ is inconsistent

- $max\,(\mathcal{A}_{\mathcal{R}'}) \leq max\,(\mathcal{A}_{\mathcal{R}'})$.

Intuitively, a maximal assertional base repartition of the ABox $\mathcal{A}_{\oplus}$ must contains all membership assertions having maximal weights, since we consider redundancy as a means of strengthening and confirmation. Note that if we ignore redundancy by omitting weights from the assertional base $\mathcal{A}_{\oplus}$, we find the classical definition of assertional-based reparation.

4.3 Redundancy in assertional-based removed set

To restore consistency, another method based on cardinality criterion has been proposed in the literature. Such strategy is based on inconsistency minimization and consists in determining smallest subsets of assertional facts that should be dropped from the current ABox in order to restore its consistency with the TBox. This approach refers to the so-called Removed Sets Revision, also known as a lexicographic-based approach, proposed in [25, 7] for revising a set of propositional formulas. It stems from removing a minimal number of formulas, called removed set, to restore consistency. The minimality in removed sets revision refers to the cardinality criterion and not to the set-inclusion criterion. Recently an extension of Removed Sets Revision within *DL-Lite* framework was proposed in [6]. More formally, a removed set is defined as follow:

Definition 10. Let $\mathcal{K} = \langle \mathcal{T}, \mathcal{A} \rangle$ be an inconsistent knowledge base. A removed set, denoted by R, is a set of membership assertions such that:

- $R \subseteq \mathcal{A}$,
- $\langle \mathcal{T}, (\mathcal{A} \backslash R) \rangle$ is consistent,
- $\forall R' \subseteq \mathcal{A}$, if $\langle \mathcal{T}, (\mathcal{A} \backslash R') \rangle$ is consistent then $|R| < |R'|$.

We denote by $\mathcal{R}(\mathcal{K})$ the set of the removed sets of $\mathcal{K}$. Note that if $\mathcal{K}$ is consistent then $\mathcal{R}(\mathcal{K}) = \emptyset$.

Definition 11. Let $\mathcal{K} = \langle \mathcal{T}, \mathcal{A} \rangle$ be an inconsistent *DL-Lite* knowledge base. A sub-base M of A is said to be a maximal consistent assertional base with $\mathcal{T}$, by cardinality criterion if and only if:

- $\{\mathcal{T} \cup M\}$ is consistent and
- there exists no M' such that $\{\mathcal{T} \cup M'\}$ is consistent and $|M'| > |M|$.

Intuitively, adding any assertional fact ϕ from $\mathcal{A} - M$ to M entails the inconsistency of $\{\mathcal{T} \cup M \cup \{\phi\}\}$. From the cardinality criterion and the definition of a Removed Set (Definition 10), the following definition holds.

Definition 12. Let M be a maximal consistent assertional base, , by cardinality criterion, with $\mathcal{T}$. Then the set $R = \mathcal{A} \setminus M$ is a removed set.

This definition simply states that a maximal consistent assertional base by cardinality criterion, is simply obtained by dropping a removed set from the assertional base $\mathcal{A}$.

Now we study the effect of redundancy in restoring inconsistency using removed sets. By considering the redundancy of an assertional fact encoded in the multi-set $\mathcal{A}_{\oplus}$ as means of confirmation the following proposition holds.

Proposition 2. *Let* (ϕ_i, n_i) and (ϕ_j, n_j) be two assertional facts involved in conflict where $n_i > n_j$. *Then* $(\phi_J, n_J) \in R(\mathcal{K})$ and $(\phi_i, c_i) \notin R(\mathcal{K})$.

This proposition simply stats that for two assertional facts involved in conflicts, the one less confirmed by sources is more plausible to be throw out that the other one.

As mentioned in Definition 7, another form of redundancy may exists. It concerns the implication of an assertional fact in conflicts. Hence the following proposition holds.

Proposition 3. *Let* (ϕ_i, c_i) and (ϕ_j, c_j) be two assertional facts involved in conflict where $c_i > c_j$. *Then* $(\phi_i, c_i) \in R(\mathcal{K})$ and $(\phi_j, c_j) \notin R(\mathcal{K})$.

This proposition simply states that is more credible to throw out from the original assertional base an assertional fact which is involved more than another in conflicts. From Propositions 2 and 3, when two assertional facts are involved in inconsistency, the preference is given to the one which is confirmed by more sources and less appearing in conflicts. Let $R \subseteq \mathcal{A}_{\oplus}$ be a removed set of $\mathcal{A}_{\oplus}$, we denote by

$$maxc\,(R) = max\,\{c_i : (\phi_i, c_i) \in R\}$$

$$maxn\,(R) = max\,\{n_i : (\phi_i, n_i) \in R\, and\, \phi_i \notin ind\,(\mathcal{A}_{\oplus})\}$$

we provide a new definition of a removed set.

Definition 13. Let $\mathcal{K} = \langle \mathcal{T}, \mathcal{A} \rangle$ be an inconsistent knowledge base. A removed set, denoted by R, is a set of membership assertions such that:

- $R \subseteq \mathcal{A}_{\oplus}$,
- $\langle \mathcal{T}, (\mathcal{A}_{\oplus} \backslash R) \rangle$ is consistent,
- $\forall R' \subseteq \mathcal{A}_{\oplus}$, if $\langle \mathcal{T}, (\mathcal{A}_{\oplus} \backslash R') \rangle$ is consistent then $|R| < |R'|$.
- $\forall R'' \subseteq \mathcal{A}_{\oplus}$, if $|R''| = |R|$ then $maxc\,(R'') \leq maxc\,(R)$ and $maxn\,(R') \geq maxn\,(R)$

Intuitively, this definition states that a removed set of must contains assertional facts less confirmed by sources and more involved in conflicts that others.

5 Conclusion

In this research report, we studied assertional-based fusion within *DL-Lite* knowledge base setting. We have shown that aggregating several sets of data provided by different sources of information comes down to finding assertional-based reparations in order to solve conflicts between sources. We first introduced the notion of assertional-based reparations based on both cardinality criterion an set inclusion criterion. Then, we studied the impact of redundant data on the definition of assertional-based reparation.

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