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Min-based assertional merging approach for prioritized *DL-Lite* knowledge bases

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Abstract

DL-Lite is a powerful and tractable family of description logics specifically tailored for applications that use huge volumes of data. In many real world applications, data are often provided by several and potentially conflicting sources of information having different levels of priority. Possibility theory offers a very natural framework to deal with ordinal and qualitative uncertain beliefs or prioritized preferences. Thus, to encode prioritized assertional facts, a possibility *DL-Lite* logic is more suited.

We propose in this paper a *min*-based assertional merging operator for possibilistic *DL-Lite* knowledge bases. We investigate in particular the situation where the sources share the same terminological base. We present a syntactic method based on conflict resolution which has a meaningful semantic counterpart when merging possibility distributions. We finally provide an analysis in the light of a new set of postulates dedicated to uncertain *DL-Lite* merging.

1 Introduction

Description Logics (DLs) provide a powerful formalism for representing and reasoning on ontologies [2]. A DL knowledge base is formed by a terminological base, called TBox, and an assertional base, called ABox. The TBox contains intentional (or generic) knowledge of the application domain whereas the Abox stores data (or individuals or constants) that instantiate terminological knowledge. In the last years, there has been an increasingly interest in Ontology-based Data Access (OBDA), in which a TBox is used to reformulate posed queries to offer a better access to the set of data encoded in the ABox [25]. Recently, a lot of attention was given to *DL-Lite*, a family of lightweight DLs specifically fitted towards OBDA [13]. *DL-Lite* is especially dedicated for applications that use huge volumes of data, in which query answering is the most important reasoning task. *DL-Lite* offers a very low computational complexity for the reasoning process. In particular query answering is in *LogSpace* for spatial complexity (w.r.t. the overall size of the ABox). Moreover knowledge base consistency test and all DLs standard reasoning services are polynomial for combined complexity (w.r.t. the overall size of the knowledge base) [1].

In many real OBDA applications, assertional facts are often provided by several and potentially conflicting sources of information having different reliability levels. Moreover, a given source may provide its set of data with different confidence levels. Possibilistic theory (*e.g.* [17]) offers a very natural framework to deal with ordinal and qualitative uncertain beliefs or prioritized preferences. This framework allows to deal with non-probabilistic information and is particularly appropriate when the uncertainty (or priority) scale only reflects a priority relation between different pieces of information. An important problem that arises in such a situation is how to aggregate these different sets of data. This problem is closely related to the *belief merging problem* (*e.g.* [12, 22]), largely studied when knowledge bases are encoded in propositional logic framework. Belief merging focuses on aggregating pieces of information issued from distinct, and possibly conflicting or inconsistent, sources of information. This process produces a global point of view over considered problems by taking advantage of pieces of information provided by each source. Within the possibility theory framework, several merging operators (*e.g.* [15, 8, 10]) have been proposed for merging pieces of information. These merging operators lead to combine multiple possibility distributions, that encode information provided by different sources, to obtain a unique possibility distribution which represents the global point of view from available information. Syntactic counterparts have been introduced for most of them.

Recently, a possibilistic extension of *DL-Lite*, denoted *DL-Lite*$^\pi$, was proposed in [3]. In particular, *DL-Lite*$^\pi$ guarantees a computational complexity that is identical to the one of standard *DL-Lite*. In this paper, we use *DL-Lite*$^\pi$ to encode and reason with available knowledge. Merging possibilistic DLs knowledge bases has been recognized as an important issue [26]. Recently, in [5], a *min*-based merging operator dedicated to possibilistic *DL-Lite* knowledge bases was proposed as an adaptation of the well-known idempotent conjunctive operator lastly introduced within possibilistic logics setting. This latter, suitable when sources are assumed to be dependent, is very cautious in the sense where it leads to ignore too many axioms in order to ensure the consistency of the resulting knowledge base.

In this paper, we go one step further in the definition of merging operators for *DL-Lite*$^\pi$ knowledge bases by investigating the aggregation of assertional bases (ABox) which are linked to the same terminological base. The rest of this paper is organized as follows. Section 2 gives brief preliminaries on *DL-Lite*$^\pi$ as extension of *DL-Lite* within possibility theory setting. In Section 3, we first introduce a syntactic merging operator, namely a *min*-based assertional operator based on conflict resolution. We show that such a merging operator gives a more satisfactory result compared with the one proposed in [5]. We then study, in Section 4, merging at a semantic level, and we show that our operator has a natural counterpart when combining several possibility distributions. We also rephrase within *DL-Lite* framework the set of postulates proposed in [23] to characterize the logical behavior of belief bases merging operators. Thus, we provide a postulates-based logical analysis of the *min*-based assertional operator in the light of this new set of postulates dedicated to the uncertain *DL-Lite* framework. Section 5 concludes the paper. Two important results of this study are: (i) our merging approach based on conflict resolution can be easily extended to define others merging operators and (ii) the computational complexity of *min*-

based assertional fusion outcome is polynomial.

2 Possibilistic DL-Lite

In this section, we recall main notions of possibilistic *DL-Lite* framework [3], denoted by $DL\text{-}Lite^{\pi}$, as an extension of *DL-Lite* within a possibility theory setting. $DL\text{-}Lite^{\pi}$ provides an excellent mechanism to deal with uncertainty and to ensure reasoning under inconsistency while keeping a computational complexity identical to the one used in standard *DL-Lite*.

2.1 A brief refresh on *DL-Lite*

For the sake of simplicity, we only present $DL\text{-}Lite_{core}$ the core fragment of all the *DL-Lite* family [13]. However, results of this paper are valid for $DL\text{-}Lite_{R}$ and $DL\text{-}Lite_{F}$, the two main fragments of the *DL-Lite* family.

A *DL-Lite* knowledge base $\mathcal{K}=\langle\mathcal{T},\mathcal{A}\rangle$ is composed of a set of atomic concepts (*i.e.* unary predicates), a set of atomic roles (*i.e.* binary predicates) and a set of individuals (*i.e.* constants). Complex concepts and roles are built as follows:

$$B \longrightarrow A|\exists R \qquad R \longrightarrow P|P^{-} \qquad C \longrightarrow B|\neg B$$

where A (*resp.* P) is an atomic concept (*resp.* role). B (*resp.* C) is called basic (*resp.* complex) concept and role R is called basic role. The TBox $\mathcal{T}$ includes a finite set of inclusion assertions of the form $B \sqsubseteq C$ where B and C are concepts. The ABox $\mathcal{A}$ contains a finite set of assertions on atomic concepts and roles of the form $A(a)$ and $P(a,b)$ where a and b are two individuals.

The semantics of *DL-Lite* is given by an interpretation $I=(\Delta,.^{I})$ which consists of an infinite and non-empty domain, denoted Δ, and an interpretation function, denoted $.^{I}$. The function $.^{I}$ associates with each individual a an element a^{I} of Δ^{I}, to each concept C a subset C^{I} of Δ^{I} and to each role R a binary relation R^{I} over Δ^{I}. Furthermore, the interpretation function $.^{I}$ is extended in a straightforward way for complex concepts and roles as follows: $(\neg B)^{I}=\Delta^{I} \setminus B^{I}$, $(P^{-})^{I}=\{(y,x) \in \Delta^{I} \times \Delta^{I}|(x,y) \in P^{I}\}$ and $(\exists R)^{I}=\{x \in \Delta^{I}|\exists y \in \Delta^{I}\, such\, that\, (x,y) \in R^{I}\}$.

An interpretation I is said to be a model of an inclusion assertion $B \sqsubseteq C$, denoted by $I \vDash B \sqsubseteq C$, iff $B^{I} \subseteq C^{I}$. Similarly, we say that an interpretation I is a model of a membership assertion $A(a)$ (*resp.* $P(a,b)$), denoted by $I \vDash A(a)$ (*resp.* $I \vDash P(a,b)$), iff $a^{I} \in A^{I}$ (*resp.* $(a^{I},b^{I}) \in P^{I}$). I is a model of $\mathcal{K}=\langle\mathcal{T},\mathcal{A}\rangle$, denoted by $I \vDash \mathcal{K}$, iff $I \vDash \mathcal{T}$ and $I \vDash \mathcal{A}$ where $I \vDash \mathcal{T}$ (*resp.* $I \vDash \mathcal{A}$) means that I is a model of all axioms in $\mathcal{T}$ (*resp.* $\mathcal{A}$). A knowledge base $\mathcal{K}$ is said to be consistent if it admits at least one model, otherwise $\mathcal{K}$ is said to be inconsistent. A *DL-Lite* TBox $\mathcal{T}$ is said to be incoherent if there exists at least a concept C such that for each interpretation I which is a model of $\mathcal{T}$, we have $C^{I}=\emptyset$. Note that within a *DL-Lite* setting, the inconsistency problem is always defined with respect to some ABox since a TBox may be incoherent but never inconsistent.

2.2 Possibility distribution over *DL-Lite* interpretation

Let Ω be a universe of discourse composed by a set of *DL-Lite* interpretations (I=$(\Delta, .^I) \in \Omega$). The semantic counterpart of a *DL-Lite*$^\pi$ knowledge base is given by a possibility distribution, denoted by π, which is a mapping from Ω to the unit interval $[0,1]$ that assigns to each interpretation $I \in \Omega$ a possibility degree $\pi(I) \in [0,1]$ that represents its compatibility or consistency with respect to the set of available knowledge. When $\pi(I)$=0, we say that I is impossible and it is fully inconsistent with the set of available knowledge, whereas when $\pi(I)$=1, we say that I is totally possible and it is fully consistent with the set of available knowledge. For two interpretations I and I', when $\pi(I) > \pi(I')$ we say that I is more consistent or more preferred than I' *w.r.t* available knowledge. Lastly, π is said to be normalized if there exists at least one totally possible interpretation, namely $\exists I \in \Omega, \pi(I)$=1, otherwise, we say that π is sub-normalized. The concept of sub-normalization reflects the presence of conflicts in the set of available information.

Given a possibility distribution π defined on a set of interpretations Ω, one can define two measures on a *DL-Lite* axiom φ: A possibility measure $\Pi(\varphi)$=$\underset{I\in\Omega}{max}\{\pi(I) : I \models \varphi\}$ that evaluates to what extent an axiom φ is compatible with the available knowledge encoded by π and a necessity measure $N(\varphi)$=$1 - \underset{I\in\Omega}{max}\{\pi(I) : I \not\models \varphi\}$ that evaluates to what extent φ is certainty entailed from available knowledge encoded by π.

2.3 *DL-Lite*$^\pi$ knowledge base

Let $\mathcal{L}$ be a *DL-Lite* description language, a *DL-Lite*$^\pi$ knowledge base is a set of possibilistic axioms of the form (φ, α) where φ is an axiom expressed in $\mathcal{L}$ and $\alpha \in \,]0,1]$ is the degree of certainty of φ. Namely, a *DL-Lite*$^\pi$ knowledge base $\mathcal{K}$ is such that $\mathcal{K}$=$\{(\varphi_i, \alpha_i) : i = 1, ..., n\}$. Only somewhat certain information are explicitly represented in a *DL-Lite*$^\pi$ knowledge base. Namely, axioms with a null degree ($\alpha = 0$) are not explicitly represented in the knowledge base. The weighted axiom (φ, α) means that the certainty degree of φ is at least equal to α (namely $N(\varphi) \geq \alpha$). A *DL-Lite*$^\pi$ knowledge base $\mathcal{K}$ will also be represented by a couple $\mathcal{K}$=$\langle\mathcal{T}, \mathcal{A}\rangle$ where both elements in $\mathcal{T}$ and $\mathcal{A}$ may be uncertain. It is important to note that, if we consider all $\alpha_i = 1$ then we found a classical *DL-Lite* knowledge base: $\mathcal{K}^*$=$\{\varphi_i : (\varphi_i, \alpha_i) \in \mathcal{K}\}$.

Given $\mathcal{K}$=$\langle\mathcal{T}, \mathcal{A}\rangle$ a *DL-Lite*$^\pi$ knowledge base, we define the α-cut of $\mathcal{K}$ (*resp.* $\mathcal{T}$ and $\mathcal{A}$), denoted by $\mathcal{K}_{\geq\alpha}$ (*resp.* $\mathcal{T}_{\geq\alpha}$ and $\mathcal{A}_{\geq\alpha}$), the subbase of $\mathcal{K}$ (*resp.* $\mathcal{T}$ and $\mathcal{A}$) composed of axioms having weights at least greater than α. We say that $\mathcal{K}$ is consistent if the standard knowledge base obtained from $\mathcal{K}$ by ignoring the weights associated with axioms is consistent. In case of inconsistency, we attach to $\mathcal{K}$ an inconsistency degree. The inconsistency degree of a *DL-Lite*$^\pi$ knowledge base $\mathcal{K}$, denoted by $Inc(\mathcal{K})$, is syntactically defined as follow: $Inc(\mathcal{K})$=$max\{\alpha$:$\mathcal{K}_{\geq\alpha}$ is inconsistent$\}$.

Given a *DL-Lite*$^\pi$ knowledge base $\mathcal{K}$, one can associate to $\mathcal{K}$ a joint possibility distribution, denoted by $\pi_\mathcal{K}$, defined over the set of all interpretations I=$(\Delta, .^I)$ by associating to each interpretation its level of consistency with the set of available knowledge, that is, with $\mathcal{K}$. Namely:

Definition 1. *The possibility distribution induced from a DL-Lite$^{\pi}$ is defined as follows:* $\forall I \in \Omega : \pi_{\mathcal{K}}(I) = \begin{cases} 1 \;\; if \, \forall \, (\varphi_i, \alpha_i) \in \mathcal{K}, I \models \varphi_i \\ 1 - max\{\alpha_i : (\varphi_i, \alpha_i) \in \mathcal{K} I \not\models \varphi_i\} \; otherwise \end{cases}$

A *DL-Lite*$^{\pi}$ knowledge base $\mathcal{K}$ is said to be consistent if its joint possibility distribution $\pi_{\mathcal{K}}$ is normalized. If not, $\mathcal{K}$ is said to be inconsistent and its inconsistency degree is defined semantically as follow: $Inc(\mathcal{K}){=}1 - \max_{I \in \Omega}\{\pi_{\mathcal{K}}(I)\}$.

It was shown in [3] that computing the inconsistency degree of a *DL-Lite*$^{\pi}$ knowledge base comes from the extension of the algorithm presented in [13] by modifying it to query for individuals with a given certainty degree.

Example 1. *Let* $\mathcal{K}{=}\langle \mathcal{T}, \mathcal{A} \rangle$ *be a DL-Lite*$^{\pi}$ *knowledge base where* $\mathcal{T}{=}\{(A \sqsubseteq B, 1), (B \sqsubseteq \neg C, .9)\}$ *and* $\mathcal{A}{=}\{(A(a), .6), (C(b).5)\}$. *The possibility distribution* $\pi_{\mathcal{K}}$ *associated to* $\mathcal{K}$ *is computed using Definition 1 as follows where* $\Delta{=}\{a, b\}$*:*

I	$.^{I}$	$\pi_{\mathcal{K}}$
I_1	A={a},B={},C={b}	0
I_2	A={a},B={a},C={b}	1
I_3	A={},B={},C={a,b}	.4
I_4	A={a,b},B={a,b},C={}	.5

Table 1: Example of a possibility distribution induced from a *DL-Lite*$^{\pi}$ KB

One can observe that $\pi_{\mathcal{K}}(I_2){=}1$ *meaning that* $\pi_{\mathcal{K}}$ *is normalized, and thus* $\mathcal{K}$ *is consistent.* □

3 Syntactic merging of *DL-Lite*$^{\pi}$ assertional bases

Let us consider $\mathcal{A}_1$,...,$\mathcal{A}_n$ a set of assertional bases (ABox) where each $\mathcal{A}_i$ represents assertional facts provided by a single source of information. We assume that we have a well-formed and coherent terminological base (TBox) $\mathcal{T}$ where each $\mathcal{A}_i$ is consistent with $\mathcal{T}$. This is not a restriction. This particular case can be handled outside the fusion problem considered in this paper. Note that this choice is motivated by the fact that such situation is widely occurring in Ontology-Based Data Access. Throughout the rest of this paper, we cast available information within the *DL-Lite*$^{\pi}$ framework. For the sake of simplicity, we omit the weights notation attached to the TBox axioms considered as the ones having the highest certainty level, namely, an axiom in $\mathcal{T}$ is of the form (φ,1). We only represent explicitly weights attached to $\mathcal{A}_i$ assertions. An assertion f in $\mathcal{A}_i$ is of the form $f{=}(\varphi, \alpha)$ where $\alpha \in [0, 1]$. Note that copies of the same assertions φ are allowed in several $\mathcal{A}_i$ and they are considered as different in the sense of priorities or certainty and not in terms of interpretations since we use the *unique name assumption*. In this section, we study syntactic merging of n assertional bases $\mathcal{A}_1$,...,$\mathcal{A}_n$ that are linked to the same TBox $\mathcal{T}$.

Let us consider $S_1, ..., S_n$ be the signatures of $\mathcal{A}_1$,...,$\mathcal{A}_n$ and $\mathcal{T}$. Recall that a signature S of a knowledge base $\mathcal{K}$ is the set of concept names and role names used in $\mathcal{K}$. We assume that all $\mathcal{A}_i$'s and $\mathcal{T}$ share the same signature. Namely if a concept name (*resp.* role name) A appears in S_1 and S_2 then A is assumed to be the same. We look to identify a syntactical merging operator on the $\mathcal{A}_i$'s *w.r.t* a TBox $\mathcal{T}$ which will be semantically meaningful. Merging at semantic level will be presented in Section 4.

3.1 Merging using the classical *min*-based operator

In this section, we perform merging of $\mathcal{A}_1$,...,$\mathcal{A}_n$ a set of ABox *w.r.t* a TBox $\mathcal{T}$ using the classical *min*-based merging operator proposed in [5] to aggregate *DL-Lite*$^\pi$ knowledge bases. This operator is a direct extension of the well-known idempotent conjunctive operator (*e.g.* [8]) within possibilistic *DL-Lite* setting. It is recommended when distinct sources that provide information are assumed to be dependent.

Let $\mathcal{T}$ be a TBox and $\mathcal{A}_1, ...\mathcal{A}_n$ be a set of ABox provided by n distinct sources of information to be linked to $\mathcal{T}$. The *min*-based merging operator, denoted by $\oplus$ considers the union of all ABox. Namely:

$$\mathcal{A}_\oplus = \mathcal{A}_1 \cup \mathcal{A}_2 \cup \ldots \cup \mathcal{A}_n.$$

The merging of two consistent knowledge bases is not guaranteed to be consistent. Namely, the resulting knowledge base $\mathcal{K}_\oplus = \langle \mathcal{T}, \mathcal{A}_\oplus \rangle$ may be inconsistent. To restore the consistency of the resulting knowledge base a normalization step is required. The following definition gives the formal logical representation of the normalized knowledge base.

Definition 2. *Let $\mathcal{T}$ be a TBox and $\mathcal{A}_\oplus$ be the aggregation of $\mathcal{A}_1, ...\mathcal{A}_n$, n ABox using classical min-based operator. Let x=Inc($\langle \mathcal{T}, \mathcal{A}_\oplus \rangle$). Then, the normalized knowledge base, denoted, $\mathcal{K}_{N\oplus}$ is such that:*

$$\mathcal{K}_{N\oplus} = \langle \mathcal{T}, \{(\varphi, \alpha) : (\varphi, \alpha) \in \mathcal{A}_\oplus \, and \, \alpha > x\} \rangle$$

Example 2 (continued)**.** *Let us continue with the TBox $\mathcal{T}$={$A \sqsubseteq B$, $B \sqsubseteq \neg C$} presented in Example 1 while assuming that the certainty degree of each axioms is set to 1. Let us consider the following set of ABox to be linked to $\mathcal{T}$: $\mathcal{A}_1$={$(A(a), .6)$, $(C(b), .5)$}, $\mathcal{A}_2$={$(C(a), .4)$, $(B(b), .8)$, $(A(b), .7)$} and $\mathcal{A}_3$={$(A(b), .2)$, $(A(c), .5)$, $(B(c), .4)$}. We have $\mathcal{A}_\oplus$={$(A(a),.6)$, $(C(b),.5)$, $(C(a), .4)$, $(B(b),.8)$, $(A(b),.7)$, $(A(b),.2)$, $(A(c), .5)$, $(B(c),.4)$} where Inc($\langle \mathcal{T}, \mathcal{A}_\oplus \rangle$)=.5. Then $\mathcal{K}_{N\oplus}$=$\mathcal{T}$ $\cup${$(A(a),.6)$,$(B(b), .8)$, $(A(b),.7)$}.* □

According to Definition 2, merging operation does not modify the certainty degrees of the *DL-Lite*$^\pi$ knowledge base. It just permits to ignore the presence of contradictions (or conflicts) and maintain all the assertions of $\mathcal{A}_\oplus$ whose certainty degrees are higher than the inconsistency degree of $\langle \mathcal{T}, \mathcal{A}_\oplus \rangle$. It is clear that the formal expression of the normalized *DL-Lite*$^\pi$ knowledge base $\mathcal{K}_\oplus$ given in Definition 2 provides a consistent knowledge base. However, this result is not very satisfactory, since many assertions in $\mathcal{A}_1$,...,$\mathcal{A}_n$, which are not involved in any conflict are thrown out. As pointed in [7], restoring consistency in possibilistic logics suffers generally from an important drawback problem in the sense that some axiom from $\mathcal{A}_\oplus$-$\mathcal{A}_{\oplus_{>Inc(\mathcal{T}\cup\mathcal{A}_\oplus)}}$ that are not involved in any conflict are inhibited as we can see in the above example.

Example 3 (continued)**.** *One can see that the assertions* $(A(c), .5)$ *and* $(B(c), .4)$ *are not involved in any conflict, but they are nor integrated in the merging result.* □

In the next section, we investigate a new approach to merge assertional base based on conflict detection. This approach allows recovering of all elements, non involved in any conflict and inhibited when restoring consistency using the classical *min*-based merging operator.

3.2 Min-based assertional merging using conflict resolution

Let $\mathcal{K}=\langle\mathcal{T},\mathcal{A}\rangle$ be a $DL\text{-}Lite^{\pi}$ knowledge base. In [3] it was shown that computing the inconsistency degree of $\mathcal{K}$ comes down to compute the one of $\langle\pi - neg(\mathcal{T}),\mathcal{A}\rangle$ where π-$neg(\mathcal{T})$ is the negated closure of $\mathcal{T}$. The negated closure will contain all the possibilistic negated axioms of the form $(B_1\sqsubseteq\neg B_2,\alpha)$ that can be derived from $\mathcal{T}$. The set π-$neg(\mathcal{T})$ is obtained by applying a set of three rules that extend the ones defined in standard *DL-Lite*. For instance after adding all NI of $\mathcal{T}$ to π-$neg(\mathcal{T})$ a rule said that If $(B_1\sqsubseteq B_2,\alpha_1)\in\mathcal{T}$ and $(B_2\sqsubseteq\neg B_3,\alpha_2)$ in π-$neg(\mathcal{T})$ then add $(B_1\sqsubseteq\neg B_3,min(\alpha_1,\alpha_2))$ to π-$neg(\mathcal{T})$. See [3] for a more detailed description of $DL\text{-}Lite^{\pi}$. Indeed, computing inconsistency degree of $\mathcal{K}$ consists on calculating the maximal weight attached to minimal inconsistent subsets involved in inconsistency. More formally, a minimal inconsistent set is defined as follows.

Definition 3. *A minimal inconsistent subset* $\mathcal{M}\subseteq\mathcal{K}$ *is a subset of* $\langle\pi - neg(\mathcal{T})$, $\mathcal{A}\rangle$ *of the form:* $\{(B_1\sqsubseteq\neg B_2,\alpha_1),(B_1(a),\alpha_2),(B_2(a),\alpha_3)\}$ *where* $(B_1\sqsubseteq\neg B_2,\alpha_1)\in$ π-$neg(\mathcal{T})$, $(B_1(a),\alpha_2)\in\mathcal{A}$ *and* $(B_2(a),\alpha_3)\in\mathcal{A}$.

Clearly, a minimal inconsistent subset is a subset of information involving three elements: an axiom of π-$neg(\mathcal{T})$ and two assertions of $\mathcal{A}$ up to a particular case where B_1=B_2 belongs to π-$neg(\mathcal{T})$. This corresponds to the situation of insatisfiable concept. Namely, no way to find an individual that belongs to B. In this case B_1=B_2 is minimal inconsistent subset composed only of two elements: an axiom of π-$neg(\mathcal{T})$ and an assertions of $\mathcal{A}$. Within a *DL-Lite* setting, the inconsistency problem is always defined with respect to some ABox, since a TBox may be incoherent but never inconsistent. Recall that in this paper, we assume that $\mathcal{T}$ is coherent. So, from the definition of minimal inconsistent subset, we define the notion of conflict as a minimal inconsistent subset of assertions that contradict a negative inclusion axiom. More formally:

Definition 4. *Let* $\mathcal{K}=\langle\mathcal{T},\mathcal{A}\rangle$ *be an inconsistent* $DL\text{-}Lite^{\pi}$ *knowledge base where axioms in* $\mathcal{T}$ *are set to 1. A sub-base* $\mathcal{C}\subseteq\mathcal{A}$ *is said to be an assertional conflict set of* $\mathcal{K}$ *iff*

- $Inc(\langle\mathcal{T},\mathcal{C}\rangle) > 0$ *and*
- $\forall\ f\in\mathcal{C}$, $Inc(\langle\mathcal{T},\mathcal{C}-\{f\}\rangle)$=0 *with* f=(φ,α)

It is clear that in Definition 4, removing any assertion φ from $\mathcal{C}$ restores the consistency of $\langle\mathcal{T},\mathcal{C}\rangle$. Recall that when the TBox is coherent, a conflict involves exactly two assertions.

Example 4 (Example continued)**.** *Let us consider $\mathcal{T}$ and $\mathcal{A}_{\oplus}$ from the above example. The π-neg($\mathcal{T}$)=$\{A\sqsubseteq\neg C,\, B\sqsubseteq\neg C\}$. One can compute the following conflict sets: $\mathcal{C}_1$=$\{(A(a),.6),(C(a),.4)\}$, $\mathcal{C}_2$=$\{(C(b),.5),(B(b),.8)\}$, $\mathcal{C}_3$=$\{(C(b),.5),(A(b),.7)\}$ and $\mathcal{C}_4$=$\{(C(b),.5),(A(b),.2)\}$.*

Let us assume that $\mathcal{A}_1$,...,$\mathcal{A}_n$ are assertional bases provided by n sources of information to be linked to the same TBox $\mathcal{T}$ and they use the same scale to represent uncertainty. Let denote by f=(φ,α) an assertion or a fact in $\mathcal{A}_i$, we define the notion of conflict vector as follows:

Definition 5. *Let $\mathcal{T}$ be a TBox and $\mathcal{A}_1$,...,$\mathcal{A}_n$ be a set of ABox provided by n distinct sources of information to be linked to $\mathcal{T}$. Then $\forall f\in\mathcal{A}_i$ we define a conflict vector associated with f=$(\varphi,\alpha)\in\mathcal{A}_i$:$\mathcal{V}(f)$=$\langle\nu_1,\nu_2,...,\nu_n\rangle$ such that*

$$\forall j = 1..n : \mathcal{V}_j(f) = \begin{cases} 1 \;\; if \; \langle\mathcal{T},\{(\varphi,1)\cup\mathcal{A}_i\}\rangle \; is\, consistent \\ Inc(\langle\mathcal{T},\{(\varphi,1)\cup\mathcal{A}_i\}\rangle)\, otherwise \end{cases}$$

Where $\mathcal{V}_i$ represents the i^{th} component of the vector $\mathcal{V}$.

Intuitively, for each assertion provided by an information source we built upon a vector that represents to what extend this latter contradicts other ones provided by other source. To this end, we add first the assertion with a highest prescribed level in each source and then we compute the inconsistency degree of this one. It is obvious that the conflict vector of a non conflicting assertion is equal to $\mathcal{V}(f)$=$\langle 1,1,...,1\rangle$. However assertions that are involved in conflict will have at least a ν_i strictly less than 1.

Example 5 (continued)**.** *One can obtain the following conflict vectors: $\mathcal{V}((A(a),.6))$ $=\langle 1,.6,1\rangle$,$\mathcal{V}((A(b),.7))$=$\langle .5,1,1\rangle$,$\mathcal{V}((A(b),.2))$=$\langle .5,1,1\rangle$,$\mathcal{V}((A(c),.5))$=$\langle 1,1,1\rangle$, $\mathcal{V}$ $((B(b),.8))$=$\langle .5,1,1\rangle$, $\mathcal{V}((B(c),.4))$=$\langle 1,1,1\rangle$,$\mathcal{V}((C(a),.4))$=$\langle .4,1,1\rangle$ and $\nu((C(b),$ $.5))$=$\langle 1,.2,.8\rangle$*

From now on, we give the way to aggregate assertional bases using conflict vectors attached to each assertion. Let denote by Σ the set of conflict vectors, we define the *min*-based assertional merging operators, denoted by Λ as follows:

Definition 6. *Let $\mathcal{T}$ be a TBox and $\mathcal{A}_1,\mathcal{A}_2,...,\mathcal{A}_n$ be a set of ABox provided by n sources to be linked to $\mathcal{T}$. Let Σ be the collection of conflict vectors associated to each assertion on $\mathcal{A}_i$. Then the min-based assertional merging operator, denoted by Λ, is defined on Σ as follows:*

$$\forall\mathcal{V}(f)\in\Sigma\text{: } \Lambda(f)=min\{\nu_i(f)\}$$

Let us denote by Σ_Λ, the vector resulting by *min* aggregation of conflict vectors.

Example 6 (Example continued)**.** *Σ_Λ contains the following elements: $\Lambda((A(a),.6))$ =.6, $\Lambda((A(b),.7))$=.5, $\Lambda((A(b),.2))$=.5, $\Lambda((A(c),.5))$=1, $\Lambda((B(b),.8))$ =.5, $\Lambda((B(c),$ $.4))$=1, $\Lambda((C(a),.4))$ =.4 and $\Lambda((C(b),.5))$=.2*

According to conflict vectors, one can associate to the set of assertions a new pre-order by attaching to each of them a new weight (*i.e.* $\forall(\varphi,\alpha)\in\mathcal{A}_i$:$(\varphi,\alpha)$=$(\varphi, \Lambda(f)))$. According to this new pre-order, we define the knowledge base resulting from fusion operation as follows.

Definition 7. *Let $\mathcal{T}$ be a TBox and $\mathcal{A}_1,...,\mathcal{A}_n$ be a set of n ABox to be linked to $\mathcal{T}$. Let $\mathcal{A}_\Lambda$=$\{(\varphi,\Lambda(f))$:$f=(\varphi,\alpha)\in\mathcal{A}_i$ and $\Lambda(f)\in\Sigma_\Lambda\}$. Let x=$Inc(\langle\mathcal{T},\mathcal{A}_\Lambda\rangle)$. Then the resulting knowledge base $\mathcal{K}_\Lambda$ is such that:*

$$\mathcal{K}_\Lambda=\langle\mathcal{T},\{(\varphi,\alpha):(\varphi,\alpha)\in\mathcal{A}_\Lambda \,and\, \alpha>x\}\rangle$$

Example 7 (continued)**.** *One can obtain $\mathcal{A}_\Lambda$=$\{(A(a),.6)$, $(A(b),.5)$, $(A(b),.5)$, $(A(c),1)$, $(B(b),.5)$, $(B(c),1)$, $(C(a),.4)$, $(C(b),.2)\}$ where $Inc(\langle\mathcal{T},\mathcal{A}_\Lambda\rangle$ =.4. Then $\mathcal{K}_\Lambda = T\cup\{(A(a),.6)$, $(A(b),.5)$, $(A(b),.5)$, $(A(c),1)$, $(B(b),.5)$, $(B(c), 1)\}$.*

According to Definition 7, it is clear that method based on conflict vectors is more productive that the classical definition of the *min*-based merging operator proposed in Definition 2. Note that this approach can easily propose others aggregation modes such as product-based merging or sum-based merging. The definition of this merging operator is based on a notion of conflict measure between sources of information. However, one can observe that original weights attached to assertions are lost. Regarding for instance assertion $B(c)$, it is provided by only one where its initial weight was .4. This means that $B(c)$ is not a totally reliable information. In the new knowledge base its weight is raised to 1. This can be justified by the fact that such assertion is not involved in any conflict. However when we need to an iteration process this approach may not be very useful. To overcome such limitation while preserving the same productivity of the fusion result, we propose the following definition.

Definition 8. *Let $\mathcal{T}$ be a TBox and $\mathcal{A}_1,...,\mathcal{A}_n$ be a set of n ABox to be linked to $\mathcal{T}$. Let $\mathcal{A}_\Lambda$=$\{(\varphi,\Lambda(f))$:$(\varphi,\alpha)\in\mathcal{A}_i\}$. Let x=$Inc(\langle\mathcal{T},\mathcal{A}_\Lambda\rangle)$. Then the resulting knowledge base $\mathcal{K}'_\Lambda$ is such that:*

$$\mathcal{K}'_\Lambda=\langle\mathcal{T},\{f=(\varphi,\alpha)\in\mathcal{A}_i : i\in\{1,...,n\},(\varphi,\Lambda(f))\in\mathcal{A}_\Lambda \,and\, \Lambda(f)>x\}\rangle$$

4 Semantic counterpart

Let us consider $\mathcal{A}_1$,...,$\mathcal{A}_n$ a set of assertional bases (ABox) where each $\mathcal{A}_i$ represents data of a single source of information. We assume that we have a well-formed and coherent terminological base (TBox) $\mathcal{T}$ where each $\mathcal{A}_i$ is consistent with the $\mathcal{T}$. Let π_1,...,π_n be the set of possibility distributions associated with $\mathcal{K}_1$,...,$\mathcal{K}_n$ where each $\mathcal{K}_i$=$\langle\mathcal{T},\mathcal{A}_i\rangle$. Namely each *DL-Lite*$^\pi$ knowledge base $\mathcal{K}_i$ is associated with a possibility distribution π_i which is its semantic counterpart. In this section, we investigate fusion of weighted *DL-Lite*$^\pi$ assertional bases at semantic level. We show that such merging operation is the natural semantic counterpart of the Λ merging operators (presented in Section 3) used to merge *DL-Lite*$^\pi$ ABox $\mathcal{A}_1$,...,$\mathcal{A}_n$ *w.r.t* a $\mathcal{T}$. More formally, given (π_1,...,π_n) possibility distributions associated with ($\mathcal{K}_1$,...,$\mathcal{K}_n$) *DL-Lite*$^\pi$ knowledge bases, then for the proposed operator Λ applied to aggregate $\mathcal{A}_1$,...,$\mathcal{A}_n$ *w.r.t* $\mathcal{T}$, we look for a *DL-Lite*$^\pi$ possibility distribution π_Λ constructed from the aggregation of (π_1,...,π_n) with the semantic counterpart of Λ that corresponds to the possibility distribution $\pi_{\mathcal{K}_\Lambda}$ induced from $\mathcal{K}_\Lambda$. Namely π_Λ=$\pi_{\mathcal{K}_\Lambda}$.

4.1 Min-based assertional merging of possibility distributions

Let us assume that π_1,...,π_n share the same domain of interpretations (namely Δ_1=...=Δ_n), and that all possibility distributions use the same scale to represents uncertainty. In [5], the semantic counterpart of the classical *min*-based operator or idempotent conjunctive operator, denoted by $\oplus$, was defined as a mapping from a vectors of possibility values ($\forall I\in\Omega$,$\nu(I)$=$\langle\pi_1(I),...,\pi_n(I)\rangle$) to an interval $[0,1]$ as follows: $\pi_\oplus(I)$=$min\{\nu_i(I)\}$. Generally merging two normalized possibility distributions gives an sub-normalized one. Normalizing $\pi_\oplus$ consists in maintaining only axioms having certainty degrees higher than the inconsistency degree deduced from $\pi_\oplus$. In this section, we deal with assertional bases merging at semantic level. We propose the natural semantic counterpart of the *min*-based assertional merging operator, denote Λ, presented in Section 3.2 which is based on conflict resolution. The following definition introduces the semantic definition of conflict vectors.

Definition 9. *Let $\mathcal{A}_1$,...,$\mathcal{A}_n$ be a set of ABox and π_1,...,π_n be a the set of possibility distributions induced from $\mathcal{K}_1$,...,$\mathcal{K}_n$ where each $\mathcal{K}_i$=$\langle\mathcal{T},\mathcal{A}_i\rangle$. Then $\forall f\in\mathcal{A}_i$ with f=(φ,α), we define semantically a conflict vector, denoted by $\mathcal{V}(f)$, as follows:*

$$\mathcal{V}(f)=\langle\Pi_{\pi_1}(\varphi),\Pi_{\pi_2}(\varphi),...,\Pi_{\pi_n}(\varphi)\rangle$$

where $\forall i$=1..n:$\Pi_{\pi_i}(f)$ denotes the possibility measure of φ induced from the possibility distribution π_i

Intuitively, a conflict vector associated to any ABox assertion represents to what extent this latter is compatible with available knowledge provided by each source.

Example 8 (continued)**.** *Assuming that Δ=$\{a,b,c\}$, let us consider the following possibility distributions π_1, π_2 and π_3 to be merged. Note that we have only considered interpretations model of $\mathcal{T}$.*

I	$\cdot^{\mathcal{I}}$	π_1	π_2	π_3
I_1	A={a},B={a},C={b,c}	1	.2	.5
I_2	A={b},B={b},C={a,c}	.4	1	.5
I_3	A={c},B={c},C={a,b}	.4	.2	.8
I_4	A={a,b},B={a,b},C={c}	.5	.6	.5
I_5	A={a,c},B={a,c},C={b}	1	.2	.8
I_6	A={b,c},B={b,c},C={a}	.4	1	1
I_7	A={a,b,c},B={a,b,c},C={}	.5	.6	1
I_8	A={},B={},C={a,b,c}	.4	.2	.5

Table 2: Possibility distributions induced from three knowledge bases

One can compute the following conflict vectors for each assertion:

$\mathcal{V}(A(a))$=$\langle max(1,.5,1,1), max(.2,.6,.2,.6), max(.5,.6,.8,1)\rangle$=$\langle 1,.6,1\rangle$, $\mathcal{V}(A(b))$=$\langle max(.4,.5,.5,5), max(1,.6,.1,.6), max(.5,.5,1,1)\rangle$=$\langle .5,1,1\rangle$, $\mathcal{V}(A(c))$=$\langle max(.4,1,.4,.5), max(.2,.2,1,.6), max(.8,.8,1,1)\rangle$=$\langle 1,1,1\rangle$, $\mathcal{V}(B(b))$=$\langle .5,1,1\rangle$, $\mathcal{V}(B(c))$=$\langle 1,1,1\rangle$, $\mathcal{V}(C(a))$=$\langle .4,1,1\rangle$ *and* $\mathcal{V}(C(b))$= $\langle 1,.2,.8\rangle$ *which are equal the ones computed syntactically in Example 5.* □

Let us denote by Σ the collection of conflict vectors associated to each assertion of $\mathcal{A}_i$. The next definition introduces *min*-based assertional merging operator, denoted Λ, on the conflict vectors of Σ.

Definition 10. *Let $\mathcal{A}_1$,...,$\mathcal{A}_n$ be a set of ABox and π_1,...,π_n be a the set of possibility distributions induced from $\mathcal{K}_1$,...,$\mathcal{K}_n$ where each $\mathcal{K}_i$=$\langle\mathcal{T},\mathcal{A}_i\rangle$. Let Σ the collection of conflict vectors associated to each assertion on $\mathcal{A}_i$ computed using Definition 9. Then the min-based assertional merging operator, denoted by Λ, is defined on Σ as follows: $\forall\mathcal{V}(f)\in\Sigma$:$\mathcal{V}(f)$=$\langle\Pi_{\pi_1}(\varphi),\Pi_{\pi_2}(\varphi),...,\Pi_{\pi_n}(\varphi)\rangle$,*

$$\Lambda(f)=min\{\nu_i(f)\in\mathcal{V}(f)\}$$

Let us denote by Σ_Λ, the vector resulting by *min*-based aggregation of conflict vectors.

Example 9 (Example continued)**.** *One can compute the set Σ_Λ as follow: $\Lambda((A(a),.6))$=.6, $\Lambda((A(b),.7))$=.5, $\Lambda((A(b),.2))$=.5, $\Lambda((A(c),.5))$=1, $\Lambda((B(b),.8))$ =.5, $\Lambda((B(c),.4))$=1, $\Lambda((C(a),.4))$ =.4 and $\Lambda((C(b),.5))$=.2*

From Definition 10, one can associate to each assertion a new weight that represents its compatibility with others assertions provided other sources.

Definition 11. *Let $\mathcal{A}_1$,...,$\mathcal{A}_n$ be a set of ABox and π_1,...,π_n be a the set of possibility distributions induced from $\mathcal{K}_1$,...,$\mathcal{K}_n$ where each $\mathcal{K}_i$=$\langle\mathcal{T},\mathcal{A}_i\rangle$. Then the possibility distribution π_Λ as follows:*

$$\forall I\in\Omega:\pi_\Lambda(I)=\begin{cases}1 \;\; if\,\forall\,(\varphi,\alpha)\in\mathcal{A}_i, I\models\varphi\\ 1-max\{\Lambda((\varphi,\alpha)):(\varphi,\alpha)\in\mathcal{A}_i, and\, I\not\models\varphi\}\, otherwise\end{cases}$$

where $\Lambda(\varphi_i)$ is the compatibility measure of φ_i computed using definition 10

Example 10. *From Example 3, we have (A(c),.1), (B(c),1), (A(a),.6), (A(b),.5), (B(b),.5), (C(a),.4), (C(a),.2). Then:*

I	I_1	I_2	I_3	I_4	I_5	I_6	I_7	I_8
π_Λ	0	0	.4	0	.5	.4	.6	0

Table 3: Possibility distributions resulting from assertional *min*-based merging of possibility distributions □

One can check that merging normalized possibility distributions may lead to sub-normalized possibility distribution. This is the case with our example. Indeed, we focus on the normalization problem when the use of *min*-based assertional operators *min* provides a subnormal possibility distribution.

Definition 12. *Let us consider: $h(\pi_\Lambda)$=$\max_{I\in\Omega}\{\pi_\Lambda(I)\}$. Then for every $I\in\Omega$ and*

$$h(\pi_\Lambda)>0,\ \pi_{N\Lambda}(I)=\begin{cases}1 \;\; if\,\pi_\Lambda(I)=h(\pi_\Lambda)\\ \pi_\Lambda(I)\, otherwise\end{cases}$$

Example 11 (continued). *From previous Example, we have:*

I	I_1	I_2	I_3	I_4	I_5	I_6	I_7	I_8
π_Λ	0	0	.4	0	.5	.4	.6	0
π_Λ	0	0	.4	0	.5	.4	1	0

Table 4: Normalized possibility distributions resulting from assertional *min*-based merging □

The following proposition states the equivalence between the semantic and syntactic approaches.

Proposition 1. *Let $\mathcal{A}_1$,...,$\mathcal{A}_n$ be a set of ABox and π_1,...,π_n be a the set of possibility distributions induced from $\mathcal{K}_1$,...,$\mathcal{K}_n$ where each $\mathcal{K}_i$=$\langle\mathcal{T},\mathcal{A}_i\rangle$. Then the possibility distribution*

$$\pi_{N\Lambda}(I) = \begin{cases} 1 \;\; if\, \pi_\Lambda(I) = h(\pi_\Lambda) \\ \pi_\Lambda(I)\, otherwise \end{cases}$$

is associated with

$$\mathcal{K}_\Lambda = \langle\mathcal{T}, \{(\varphi, \Lambda(f)) : (\varphi, \Lambda(f)) \in \mathcal{A}_\Lambda \, and \, \Lambda(f) > x\}\rangle$$

4.2 Logical properties

Let us use E=$\{\mathcal{K}_1, ..., \mathcal{K}_n\}$ to denote a multi-set, called belief profile, that represents the knowledge bases to be merged (where each $\mathcal{K}_i$ is associated with a possibility distribution π_i). Let us use $\triangle$ to denote a merging operator. This merging operator can be parametrized by an integrity constraint, being a konwledge base $\mathcal{K}$, and $\triangle_\mathcal{K}(E)$ denotes the result of the merging operator under this constraint $\mathcal{K}$. A logical characterization of integrity constraint merging operators has been proposed in [22] through a set of rational postulates extended from the ones proposed for belief revision [20]. The following postulates rephrase the ones proposed in [22] within *DL-Lite* framework.

($\mathbf{M_0^\pi}$) $\triangle_\mathcal{K}(E) \models \mathcal{K}$

($\mathbf{M_1^\pi}$) if $\mathcal{K}$ is consistent, then $\triangle_\mathcal{K}(E)$ is consistent

($\mathbf{M_2^\pi}$) if $\mathcal{K} \cup \bigcup_{\mathcal{K}_i \in E} \mathcal{K}_i$ is consistent, then $\triangle_\mathcal{K}(E)$=$\mathcal{K} \cup \bigcup_{\mathcal{K}_i \in E} \mathcal{K}_i$

($\mathbf{M_3^\pi}$) if $E_1 \approx E_2$ and $\mathcal{K}_1 \equiv \mathcal{K}_2$, then $\triangle_{\mathcal{K}_1}(E_1) \equiv \triangle_{\mathcal{K}_2}(E_2)$.

($\mathbf{M_4^\pi}$) if $\mathcal{K}_1 \models \mathcal{K}$ and $\mathcal{K}_2 \models \mathcal{K}$, then $\triangle_\mathcal{K}(\mathcal{K}_1 \cup \mathcal{K}_2)$ is consistent implies that $\triangle_\mathcal{K}(\mathcal{K}_1 \cup \mathcal{K}_2) \cup \mathcal{K}_2$ is consistent

($\mathbf{M_5^\pi}$) $\triangle_\mathcal{K}(E_1) \cup \triangle_\mathcal{K}(E_2) \models \triangle_\mathcal{K}(E_1 \uplus E_2)$

($\mathbf{M_6^\pi}$) if $\triangle_\mathcal{K}(E_1) \cup \triangle_\mathcal{K}(E_2)$ is consistent, then $\triangle_\mathcal{K}(E_1 \uplus E_2) \models \triangle_\mathcal{K}(E_1) \cup \triangle_\mathcal{K}(E_2)$

($\mathbf{M_7^\pi}$) $\triangle_\mathcal{K}(E) \cup \mathcal{K}' \models \triangle_{\mathcal{K} \cup \mathcal{K}'}(E)$

($\mathbf{M_8^\pi}$) if $\triangle_\mathcal{K}(E) \cup \mathcal{K}'$ is consistent, then $\triangle_{\mathcal{K} \cup \mathcal{K}'}(E) \models \triangle_\mathcal{K}(E) \cup \mathcal{K}'$

$(\mathbf{M^{\pi}_{maj}})$ $\exists n \ \triangle_{\mathcal{K}}(E_1 \uplus E_2^n) \models \triangle_{\mathcal{K}}(E_2)$

$(\mathbf{M^{\pi}_{I}})$ $\forall n \ \triangle_{\mathcal{K}}(E_1 \uplus E_2^n) \equiv \triangle_{\mathcal{K}}(E_1 \uplus E_2)$

With:

1. $\mathcal{K}_1 \models \mathcal{K}_2$ iff $\arg\max_I \pi_{\mathcal{K}_1}(I) \subseteq \arg\max_I \pi_{\mathcal{K}_2}(I)$
2. $\mathcal{K}_1 \equiv \mathcal{K}_2$ iff $\mathcal{K}_1 \models \mathcal{K}_2$ and $\mathcal{K}_2 \models \mathcal{K}_1$
3. $E_1 \approx E_2$ if and only if there exists a bijection g from E_1 to E_2 such that $\forall \mathcal{K} \in E_1 : \pi_{\mathcal{K}}{=}\pi_{g(\mathcal{K})}$
4. $\uplus$ is the union of multisets [21]
5. $E^n = \underbrace{E \uplus ... \uplus E}_{n \text{ times}}$

Note that in the special case where we only consider only one TBox $\mathcal{T}_1$ for E, these postulates are equivalent with the ones proposed in [27], by considering the revision of $\mathcal{T}_1$ by the shared TBox $\mathcal{T}$. Hence, our postulates extend (with very few adaptations) the notion of Revision of [27].

For the merging process considered in the present paper, the integrity constraint is $\mathcal{K}{=}\langle \mathcal{T}, \emptyset \rangle$ where $\mathcal{T}$ is the set of TBox axioms of each $\mathcal{K}_i \in E$ and $\mathcal{K}_i = \langle \mathcal{T}, \mathcal{A}_i \rangle$.

Proposition 2. *Our min-based assertional merging merging satisfies* $(\mathbf{M^{\pi}_{0}})$, $(\mathbf{M^{\pi}_{1}})$, $(\mathbf{M^{\pi}_{2}})$, $(\mathbf{M^{\pi}_{3}})$, $(\mathbf{M^{\pi}_{5}})$, $(\mathbf{M^{\pi}_{6}})$, $(\mathbf{M^{\pi}_{7}})$, $(\mathbf{M^{\pi}_{8}})$, $(\mathbf{M^{\pi}_{I}})$ *and falsifies* $(\mathbf{M^{\pi}_{4}})$, $(\mathbf{M^{\pi}_{maj}})$.

5 Conclusion

We propose in this paper a new operator for merging multiple sources ABoxes sharing a same terminology in the context of *DL-Lite*$^{\pi}$. We propose a syntactic version of this operator and its semantic counterpart. This operator turns out to be more productive than the operator previously proposed in [5], without increasing the complexity of the merging process. In particular, it picks any pieces of information that is not in contradiction with other bases: it is not affected by the drowning effect. We finally provide an analysis in the light of a new set of postulates dedicated to uncertain *DL-Lite* merging.

This paper opens several perspectives. For instance, we focus on a *min* operator for aggregating conflict vectors, in order to preserve possibilistic semantics. Nevertheless, other aggregation operators can be considered (e.g. the product operator) or direct comparisons from vectors (e.g. G-max based operator). From a postulate point of view, other postulates dedicated to DL knowledge bases could be studied and adapted (e.g. arbitration [23]).

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